Modern fluid motion physics
Part 1: on the static pressure law in the pipe flow element

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The solution of problem on distribution of static pressure in the real gas stream along the pipe flow element is for the first time achieved. The solution is obtained on a basis of consideration of contact interaction of the real fluid - gas and liquid - stream with the pipe wall. Physically adequate and mathematically correct expressions for the static pressure distribution are obtained by means of use of three fundamental laws in fluid dynamics: Torricelli formula, Bernoulli equation and Weissbach-Darcy formula. General solution is obtained for the gas stream. Special case of the obtained general solution is derived for the liquid stream.

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Nomenclature

D – internal diameter of the flow element;
g – gravity acceleration;
H – general height of free fall;
h_i – height of running point at free fall;
L – general metric length of the flow element;
$\bar{L}$ – general caliber length of the flow element, L/D;
l – metric length of stream from the outlet section of the flow element up to running point;
p_0, p_h – pressure at inlet and at outlet of the flow element respectively;
$p_{st}(l)$ – static pressure along the fluid stream;
V_{fv} – a stream velocity, determined by weight flow;
γ – weight density of fluid;
ζ_{in} – coefficient of local hydraulic resistance for inlet into the flow element;
ζ_{ex} – coefficient of local hydraulic resistance for outlet from the flow element;
ζ_{loc} – coefficient of local hydraulic resistance within the limits of the flow element;
λ – hydraulic friction – Darcy -- coefficient.

Introduction

Creation of physically substantial bases of fluid motion theory envisions overcoming a number of problems and solution of series of attendant questions. The problem of determination of distribution of the energy potential along the pipe flow element or system is one of fundamental

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in this area. The essence of the problem lies in the fact to define the unitized mathematical expression, which wills physically adequate and technical exactly determine the distribution of energy potential, applied to the flow element or system, and one wills be equally fairly for gas and liquid streams. In applied sense, it means determination of the law of static pressure distribution along the pipe flow element or system.

Approach

For solution of the problem, the author utilized:
- formula for free fall of solid bodies and liquid jets, determined in its preliminary kind by Torricelli -- Galilei (1643) and established in its final form by Borda – Du Buat (not later 1766);
- formula for determination of friction losses at motion of real liquid stream in a straight pipe, established by Weissbach -- Darcy (1857), and also formula for determination of losses, bound mainly with local change of a form and a cross-section area of the flow element, system, and proposed by Weissbach (1865);
- equation of the energy conservation (density of energy is more exact) for fluid stream in a pipe by Bernoulli principle (1738) in the form, that conform to the end of XIX century.

The sufficiency of these conformities to natural laws for reaching the set object is determined by their features stated below.

Torricelli – Galilei – Borda – Du Buat, TGBD, formula represent the historically first equation of the energy conservation for motion of a solid body and a fluid flow without a contact interaction. The path of development of the formula from its initial experimental form to its physically correct record has been required almost 150 years (Du Buat) and for usage it as equation of the energy conservation it has been required in addition almost 120 years. The formula has passed verification in general mechanics and then in a fluid mechanics, and it has been assumed as one of the fundamental in mechanics.

Weissbach – Darcy formula represent the historically first modification of TGBD formula for real fluid flow with taking into account of a contact interaction of the flow with a pipe wall. The formula is also obtained on a base of experimental research, and one reflects an approach to a viewing of motion, pointed by Aristotle (328 B.C.) as the third problem of his "Mechanical problems". Fundamentality of taking into account of a contact interaction, called simplistically as hydraulic friction, is affirmed not only Aristotle's prevision, but also that the formula became an inalienable part of equation of the energy conservation for a fluid flow in a pipe.

Weissbach formula is modification of Weissbach – Darcy formula, and one allows determining the different shape local resistances of a flow system to a fluid motion.

Fundamental character of equation of the energy conservation for fluid motion in a flow element or system cannot, apparently, give rise to doubt and furthermore the equation is constructed by means of the above-mentioned laws. The energy conservation principle for a movement without a contact interaction and the energy consumption principle for a movement with a contact interaction is harmonic combined in the equation. The last reflects even more severe sense, connected with self-organizing of a fluid stream, ensuring minimum energy consumptions for its motion in the flow system.

Solution

So, TGBD formula may be written in the form

$$V^2/(2g) = H(1 - h/H), \tag{1}$$

where h represents simultaneously part of not yet passed path by impinging body and part of residual potential energy, and H represents simultaneously all path of falling and available initial value of potential energy. In such aspect, formula (1) allows determining a quantity of kinetic energy of the falling body depending on relative part of a passed path of falling.

In its turn, Weissbach – Darcy formula in the form

$$p_{st}(l) \equiv \gamma h(l) = \lambda \bar{L}(1 - l/L)\gamma V_{fw}^2/(2g) + p_h \qquad (2)$$

allows determining the size of static pressure in liquid stream along the pipe depending on intensity of a contact interaction of the stream with the pipe wall. The reference point of a current length of the stream starts from the pipe outlet section, just as the reference point of a current height of a fall in the TGBD formula starts from the end point of the fall. With taking into account of denotation$(1 - l/L) = K_l$, Weissbach – Darcy formula assumes a compact form

$$p_{st}(l) \equiv \gamma h(l) = \lambda \bar{L} K_l \gamma V_{fw}^2/(2g) + p_h. \qquad (3)$$

The equation of energy conservation for liquid flow in direct pipe of round cross-section, with taking into account of formula (3) and Weissbach formula for local resistances, becomes

$$p_0 = p_h + \gamma V_{fv}^2/(2g) + (\lambda \bar{L} K_l + \zeta_{in} + \zeta_{ex} + \sum \zeta_{loc})\gamma V_{fv}^2/(2g). \qquad (4)$$

After determination V_{fv}^2 from equation (4) and substitution it in the formula (3), we receive

$$p_{st}(l) = (p_0 - p_h)\, \lambda \bar{L} K_l/(1 + \lambda \bar{L} K_l + \zeta_{in} + \zeta_{ex} + \sum \zeta_{loc}) + p_h. \qquad (5)$$

The expression (5) determines distribution of static pressure in the real compressible fluid – gas -- stream along the pipe.
For the real incompressible fluid – liquid – stream, it is valid to write the expression (4) in the kind

$$V_{fv}^2(l) = (p_0 - p_h)\, 2g/[\gamma(1 + \lambda \bar{L} K_l + \zeta_{in} + \zeta_{ex} + \sum \zeta_{loc})] = const. \qquad (6)$$

From equality (6), it is follows necessity to accept$K_l = 1$, executable by $l = 0$, what corresponds to the pipe outlet section. Therefore for liquid, the distribution 0f static pressure along the pipe has accordingly a kind

$$p_{st}(l) = (p_0 - p_h)\lambda \bar{L} K_l/(1 + \lambda \bar{L} + \zeta_{in} + \zeta_{ex} + \sum \zeta_{loc}) + p_h \qquad (7)$$

and testifies to linear change of static pressure along the liquid stream length in pipe according to a change of the fraction numerator in its right member. In contrast to this, expression (5) possesses the generality and one is applied to the gas flow in a pipe. Presence of a complex, varying along the pipe length$0 < K_l < 1$ both in numerator and in denominator of fraction in the right member of expression (5), specifies the nonlinear character of the static pressure change of a gas flow in a pipe. At the same time diminution of the static pressure in the gas flow in direction to the pipe outlet happens more and more intensive. Physically it means an increase of the gas flow velocity to the pipe outlet at its constant cross-section area and constant quantity of the hydraulic friction coefficient. Such differences of physical properties of gas from liquid as its high elastic – reversible – compressibility and high kinematic viscosity in combination with its low heat capacity stipulate consecutive transformation of a friction work to a heat, which one, in turn, stipulate consecutive increase of the gas flow velocity along the straight pipe of constant cross-section area. The common in distribution of static pressure along the pipe for liquid and gas is in that the initial and final quantities of its static pressure can be the same for these fluids. And a difference is in that, at the mentioned identity, the curve of static pressure of a gas flow between the points $p_{st}(0) = p_0$ and $p_{st}(L) = p_h$ is situated above a straight line of static pressure of a liquid flow.

In problem on free fall in gravity field, the velocity of a body is increased along the way of the fall under stationary value of gravity acceleration. In problem on the gas stream motion in a pipe, the flow velocity is increased under stationary value of intensity of a contact interaction of the gas stream with the pipe wall. Such acceleration of a gas stream is the frictional self-acceleration.

Discussion of results

The analogous object was pursued by H.A. Duxbury in his work [1]. The equation, derived by the author, contains the link of pressure drop along the pipe with the mass flow rate of fluid. The link is not immediate, because it requires introducing the discharge coefficient as the additional empirical factor, defined by means of experiment. Furthermore, the link of pressure drop with the mass flow rate is complicated by necessity of determination of the fluid density at the beginning and the end of stream as well as an average quantity in conditions of essential non-linear nature of a change of the pressure drop along the pipe. The mentioned imperfections are a consequence of the physically inferior approach, based on balance of the forces, but not an equation of the energy conservation. Therefore the equation, derived in [1] for determining the pressure drop along the pipe length as many other attempts before him, not possess the attributes of the static pressure law for the pipe flow element.

Congruence of results

The obtained expressions (5) and (7) are used as a basis for elaboration of mathematical algorithm and then VeriGas program for computing the state and motion parameters of the gas stream in diverse of the flow elements and systems, formed by its. The results of computing are tested by means of comparison with experimental data of great number of sources of the specialized literature. In particular, the diagram, adduced in the book [2], shows experimental curves of static pressure in air stream along the 80-calibers pipe, named as smooth. The analogous curves were obtained by means of the adduced here formula (5). It was found, that experimental curves exceed the calculation ones approximately on 5%. Attentive consideration of the book diagram shows that about 20 orifices were pierced in the smooth pipe, used in experiment, for the pressure measurement. There are a lot of such examples in papers and monographs [3, 4 and 5].

Final remarks

The expressions, adduced here, have the key nature; therefore their deduction is accomplished in one-dimensional stationary statement. At the same time, the results of computational experiments, that repeatedly realized, not yield to experimental results on regularity and precision.

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