Modern fluid motion physics
To the strength first problem full solution: mechanics of a necking

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Essentially new approach to analysis of internal forces, arising in cylindrical rod under action of an axial tension force, has allowed detecting the three-dimensional axisymmetric stress state. According to the new conceptual model the axial tension force causes the tangential – hoop and radial – stresses side by side with axial (normal) stress in the rod volume. Completion of Lame solution of the problem on the stress state in heavy-walled cylindrical shell has allowed ascertaining a mutual direct and reverse connection of the tangential and radial stresses with the axial stress also in the rod under action of an axial tension force. The new approach for the first time has allowed to give the full physically adequate and mathematically sufficiently strict description of a change of initial cylindrical form of a mild steel rod under action of an axial tension force on all stages of its deforming, including the necking, the fracture process and a view of the fracture surface. The new approach has allowed on the united methodological base to elucidate also a number of questions, bound with axisymmetric form of the soap solution film between two rings, with the breaking up of a liquid free jet into drops, with the causes of a buckling of the long tube under action of internal pressure, created in it by the rested and moving fluid and others.
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Introduction

One of the methods of experimental determination of physical and mechanical properties of structural metals and in the first instance of steel is the tension test of specimens with its working part in the kind of cylindrical rod. Dimensions of these specimens are standardized, and its diameter is usually 10 – 20 mm and relative length of its working part is $2.5 \leq L/D \leq 10$. The special tension-testing machines are used by the technical and research institutes and by the machine-building plants for determining of physical and mechanical properties of new structural metals and also for a control of the strength and plasticity level of machine, apparatus and its structure parts. The tension tests for determination of static strength are conducted under the limited velocity of the specimen tension. Side by side with strength, the plasticity is one of basic characteristics of steel, determining its application. Relative decrease of the specimen cross-section area at a fracture during the tension-test is its plasticity measure. The relative narrowing for the most of structural metals is usually 50 ∓ 20 % and one is connected with a decrease of the specimen working part diameter in the whole and with forming of the so-called "neck". "Cup and cone" is a name of the fracture surface in the neck zone, by professional slang. Taking into account the great importance of the plasticity for structural metals and a lack of satisfactory explanation of a change of specimen form during the tension test, author of the given article sets himself as an object creation of rational conception, which describes the behavior of the steel specimen in the kind of cylindrical rod under action of a tension force physically adequately and mathematically sufficiently strictly. One of basic requirements to the conception is, on the author opinion, a unity of approach to description of a change of the specimen form in all stages of its deforming: from a beginning of the tension test up to the specimen fracture.

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Approach

The approach basis to solution of the raised problem consists in the following wordy formula:
- longitudinal tension of cylindrical rod of isotropic material causes in it three-dimensional axisymmetric field of internal forces, which are the tangents to orthogonal assemblage of cylindrical – right- and left-handed – helices; these opposite helices are trajectories of action of internal forces in the axisymmetric rod under action of the tension force; in the electromagnetism theory such type lines are called by the field lines since M. Faraday, and in the study of strength of materials ones are called by the principal stress trajectory;
- on surface of the rod these opposite helices form a system of the longitudinal tension forces and of the transversal – hoop – tension forces of equal to each other in its quantity;
- the wall boundary layer of the rod has its cross-section area equal to a half the rod cross-section area and this layer is under action of the transversal – hoop – tension forces, increasing to its internal boundary;
- this layer, in the kind of the heavy-walled cylindrical shell, compresses the rod core;
- simultaneously both the rod core and the rod shell, compressing it, are under action of the same longitudinal tension forces, equal in a sum to the tension force, applied to the rod.

Solution

Presentation of the rod integral volume in the kind of the rod shell and the rod core, in accordance with the stated approach wordy formula, allows using G. Lame formulas (1833) for solution of the given problem by means of analysis of transversal, radial and axial stresses and pressure on a boundary of these parts of the rod.
Fig. 1 shows the diagrams of forces, stresses and pressures for cylindrical rod under action of an axial tension forces.
Beforehand, it is necessary to take notice, that transversal stresses are determined by a sum of two transversal forces according to two components – right- and left-handed – of orthogonal assemblage of cylindrical helices

$$\sigma_t^r + \sigma_t^l = 2\sigma_t, \tag{1}$$

since $\sigma_t^r = \sigma_t^l$. Every of these two opposite cylindrical helices stipulates an action of the torque in the tensioned rod in a corresponding direction: clockwise and anti-clockwise. These two equal in a quantity and acting in opposite directions torques, not destroy each other, but ones create the transversal tension stress in the tensioned rod.
Side by side with it, an orthogonality of the field lines determines also equality of transversal and axial stresses on surface of the rod

$$2\sigma_t = 2\sigma_z. \tag{2}$$

Under action of the transversal stresses the rod shell renders a pressure on its core. At the same time the rod shell, in consequence of the great quantity of a ratio of its cross-section radii

$$r_1 = r_2 \cdot \sqrt{2}/2 \cong 0.707 r_2,$$

can be attributed to the heavy-walled cylindrical shells under action of internal pressure.
In the considered case, Lame formula for determining the transversal stress on external surface of the rod shell has the kind

$$2\sigma_{t(r=r_2)} = p\frac{r_1^2}{r_2^2 - r_1^2}\left(1 + \frac{r_2^2}{r_2^2}\right) = 2p = 2\sigma_z, \tag{3}$$

i.e. the pressure quantity on a boundary of the rod shell and the rod core is equal to a quantity of the transversal stress on external surface of the rod and one is simultaneously equal to the longitudinal – axial – stress.

Transversal stress in the rod shell on a boundary with the rod core is, correspondingly,

$$2\sigma_{t(r=r_1)} = p\frac{r_1^2}{r_2^2 - r_1^2}\left(1 + \frac{r_2^2}{r_1^2}\right) = 3p \;\; or \;\; \sigma_{t(r=r_1)} = 1.5\sigma_t. \tag{4}$$

At that, radial stresses in the rod shell are changed within the limits from a zero on its external surface up to $\sigma_r = p$ on its internal surface.
Simultaneously, the well-known condition on a constancy of algebraic sum of transversal and radial stresses in a heavy-walled cylindrical shell under action of internal pressure

$$\sigma_t + (-\sigma_r) = const$$

is satisfied all over the cross-section surface of the rod shell and one confirms a constancy of longitudinal stress on this surface.
Transversal and radial stresses in the rod core, taking into consideration its compactness, has simple kind

$$\sigma_t = \sigma_r = -p = -\sigma_z$$

all over its cross-section surface.
The adduced expressions for determining of the stresses in the cylindrical rod under action of the tension force are just both on the elastic and plastic deformation stages while the rod retains cylindrical form.
At the same time the results of the numerous tension tests of specimens in the kind of cylindrical rod of plastic metals, in particular, of mild steel show that a form of such specimen is essentially changed during a process of its plastic deforming, although one remains axisymmetric. The general features of the form changes of such specimen at the plastic deforming under action of the tension force are diagrammatically presented in the article [1]. The most typical forms in the process of the specimen irreversible lengthening are the followings:
- the not great local narrowing of cylindrical part arises near one of the specimen end;
- the narrowing moves smoothly in the kind of a wave along the specimen cylindrical part as the tension force is increased and one envelops all working part of the specimen; in a result the specimen diameter becomes equal to the local narrowing diameter, mentioned in previous stage;
- the working part of the specimen takes the shape of a corset with its waist in the middle of the specimen working part;
- local neck arises in the corset waist, and one results quickly to the specimen fracture.

Geometrical analysis of the listed forms allows ascertaining that every of them can be constructed by adding only two surfaces of rotation to the initial cylindrical surface: the one-hollowed hyperboloid and an evolventoid.
The choice of the one-hollowed hyperboloid is conditioned by two basic reasons. Firstly, this surface can be considered as the cylindrical surface modification if a generatrix of the latter will be split in two crossed inclined straight lines, which ones are symmetrically inclined relatively of the rotation axis and ones connect by itself the circumferences in the cylinder bases. The very visual description of a method of transformation of cylindrical surface into the one-hollowed hyperboloid surface is adduced by prof. D. Pedoe in his beautiful book [2]. According to this method, a cylinder can be constructed by means of two same wire rings, connected each other by the parallel threads. Then a turn of one of the ring in its plane on some angle, under condition of the same tension of all threads, transforms the cylinder into the one-hollowed hyperboloid of rotation. In contrast to the cylinder of rotation, possessing only one ensemble of the generatrices, a surface of the hyperboloid contains two assemblages of generatrices. The second ensemble of generatrices can be formed by a turn of that ring from initial position in opposite direction by that angle. Naturally, a height of such hyperboloid will be less of a height of initial cylinder proportionally to the incline angle of its generatrices. Secondly, the multitude of the one-hollowed hyperboloids contains a modification, in which the crossing angle of its generatrices is equal 90^0 and simultaneously its intersection angle is 45^0 to the rotation axis. A height of such hyperboloid is equal to the chord length of the circle quarter in its bases. Thus, a height of such

hyperboloid is restricted, in contrast to a cylinder. The other method of such hyperboloid construction is in that a cube is taken as initial geometrical figure. If the upper and lower sides of this cube will be circumscribed by circles and the diagonals of one of the cube lateral sides will be rotated about vertical axis, passing through the centers of these circles, the formed surface of rotation will be present by itself the one-hollowed hyperboloid with its height, equal to its neck diameter. One of features of such hyperboloid is in that the cross-section area of its neck is equal to a half its basis area, which corresponds approximately to relative narrowing of specimens of the most of structural metals under the tension test. This special modification of the hyperboloid is called by the regular one-hollowed hyperboloid. In the context of a given problem this hyperboloid modification can be functionally called by the orthogonal one-hollowed hyperboloid of rotation. Conformably to a tension of cylindrical specimen by axial force, an influence of its form onto its stress state is in the following. While the specimen retains its cylindrical form, the orthogonal assemblage of the field lines causes transversal stresses in its shell, which compress its core by pressure, equal in a quantity to the tension stress. Such action on the specimen core promotes preservation of its continuity during the plastic flow stage up to a reaching of the tension force culmination (ultimate strength). Beginning from this moment, energy of a field of the tension forces reaches a level of infrastructural energy of the specimen material, preserving its initial form, and then one exceeds the latter. Now the orthogonal assemblage of internal forces splits a generatrix of cylindrical surface in two crossing inclined straight lines that are symmetrically intersected with the specimen axis and, in that way, one transforms the specimen cylindrical form into hyperboloid form, called by a corset. According to experimental data [1] such corset is formed by a turn of one of the base circles of initial cylinder in a beginning to an angle approximately $\mp 32^0$ and then – approximately up to $\mp 36^0$. The hyperboloidal corset is simultaneously formed in the whole length of the specimen working part and because of it the lengthening velocity of the specimen becomes commensurable with velocity of the tension-testing machine action. In a result the tension force, recorded by the machine, is decreased. A decreasing of the specimen cross-section area in the corset waist zone leads to a subsequent increasing of the specimen stress state and results to local narrowing in the kind of the orthogonal hyperboloid. Now, at last, the above mentioned cylindrical helices are straightened and coincide with the orthogonal hyperboloid generatrices. In the study of strength of materials such local narrowing is called by a neck. The neck formation is going on quickly and accompanied by ~ 50 % decrease of the specimen cross-section area. Accordingly to such transient and great transversal deformation, a lengthening of the specimen is going on again with velocity commensurable with velocity of the tension-testing machine action. The machine records again a decrease of the tension force. In the necking process, the smooth conjugation of the orthogonal hyperboloid surface with the hyperboloidal corset is formed by the rotation surface with its generatrix, possessing a variable curvature. The most acceptable curve from the spiral assemblage for such generatrix is a circumference evolvent. While the tensioned specimen retains cylindrical form, the development angle of this evolvent is equal to a zero. In the moment of a forming of the neck, before the specimen fracture, the development angle of this evolvent is $\sim 20^0$ in a point of its conjugation with the surface profile of orthogonal hyperboloid.

Fig. 2 shows diagrams of constructing of longitudinal profile of the tensioned cylindrical rod in the necking stage under action of the axial tension force only.

Fig. 3 shows, for comparison, that the development angle of the evolvent reaches the maximum quantity $54^0 44'$ under action of lateral hydrostatic pressure only.

The above-mentioned straightening of cylindrical helices and its coincidence with the orthogonal hyperboloid generatrices result to a ceasing of an acting of transversal stresses in this part of the specimen and, correspondingly, to a ceasing of a pressing of the specimen core by the specimen shell. A decrease up to ~ 50 % of the specimen cross-section area in the neck corresponds to a doubling of the axial tension stress both in sell and core parts of the specimen in its minimum cross-section. Under these conditions, the specimen shell possesses a possibility for its subsequent plastic deformation owing to a free external surface. In contrast to it, such possibility

for the specimen core is restricted by transversal (hoop) rigidity of the specimen shell. The restriction of possibility for narrowing of the specimen core in the neck zone under action of the extreme axial tension stress leads to a developing of three-dimensional tension of the specimen material in this zone. This stress state is, in the beginning, accompanied by some additional narrowing of the neck, then by appearance of a porosity of the specimen core in this zone and, at last, by developing of transversal crack from the specimen axis in cross-section, corresponding to the conjugation point of the surface profiles of the orthogonal hyperboloid and the evolventoid (point C or point E in fig. 3).The subsequent fracture of the specimen shell is going on plastically by way of a shear owing to the deformation freedom of its external surface and partially of its surface. Usually such form of the specimen fracture is called by "cup and cone." One of features of the one-hollowed hyperboloid of rotation is that a cross-section circular form of its neck is lightly transformed into an ellipse. Therefore sometimes [3] the initial transversal crack in the specimen core takes also the ellipse form. Subsequent fracture of the specimen shell is started near the ends of the ellipse major axis and one is completed by the neck-and-split fracture.
In the frames of the above stated approach, the mechanics of the local narrowing, spreaded along the specimen working part in initial stage of its plastic deformation, can be presented by the following way. In the initial state, polycrystalline structure of mild steel is statistically isotropic and one contains, by the experimental and theoretical research data [4], the rigid intracrystalline lattice of a cementite and imperfections of the crystals in themselves in the kind of dislocations [5 – 7]. In initial stage of the specimen plastic deformation, energy of the above described axisymmetric volumetric field of internal forces, excited by the axial tension force, is sufficiently only for subsequent fracture of the intracrystalline cementite lattice and for deforming of some crystals, possessing a corresponding orientation of its dislocations. As a result of spreading of the local narrowing along the specimen length, its structure is transformed into the statistically orthotropic. In the other words, initial stage of the specimen plastic deformation is a stage of volumetric structural adaptation of the specimen material to a structure of the volumetric field of internal forces, caused by an axial tension force. A surface of initial local narrowing is also a combination of the hyperboloid and evolventoid rotation.
The results of solution of the problem, stated in the given article, testify to that the pure uniaxial stressed state is possibly only for the rod with its diameter, equal a zero. At the same time, possibility of resolution of three-dimensional field of internal forces into orthogonal components is kept in full.

Discussion of results

Comparison of the approach and solution of the stated problem with results of the well-known numerous experimental researches testifies to the following.
Lueders' lines [8] testify to possibility of action of orthogonal assemblage of the field lines in the kind of the left- and right-handed helices in cylindrical specimen of mild steel under action of an axial tension force.
E. Bollenrath, V. Hauk and E. Osswald [9], using radiography method, had investigated residual stresses in cylindrical specimen of mild steel after its 11 % preliminary tension. This research allows ascertaining that the area, occupied by residual axial compressive stresses, has a form of a ring along the external contour of the specimen cross-section, this area is equal 50% the specimen cross-section area. The same area is occupied by the residual axial tension stresses in the specimen core.
S.P. Timoshenko [10] cites photos of the short ($L/D = 5.4$) thin-walled ($\delta/D = 0.07$) tubes, tensioned up to its fracture. These experiments were conducted by E.A. Davis (Westinghouse Research Laboratories) [11] and ones had showed that a change of a form of tubular specimen under action of a tension force only corresponds sufficiently to that in the case of the solid cylindrical specimen.
Among the great volume of information, dedicated to experimental researches of mechanical properties of metals by means of the solid cylindrical specimen, P.W. Bridgman's [12]

researches are brightly distinguished by combination of the tension force with lateral hydrostatic pressure. These researches had showed that an action of lateral hydrostatic pressure side by side with the tension force result to a decrease of the cross-section area of the specimen core in the neck zone. Under sufficiently high lateral pressure the specimen core disappears practically in this zone and plastic fracture is going on in the specimen shell.

V.I. Feodosjev [13] informs that a plastic deformation and fracture of cylindrical rod under action of sufficiently high lateral hydrostatic pressure in absence of the axial tension force has character and shape quite similar to a behavior of that rod under action of the axial tension force only. Using the fig. 1 *f, g*, it is easy to see that an action of lateral hydrostatic pressure allows not only compensating the tension forces between a core and a shell of the rod in the necking stage, but also ensuring a compressing of the rod by its shell. At that, the subsequent tension of the rod will be accompanied by a narrowing of the rod core into a point, similar to the above mentioned Bridgman's experiments.

Thus, the results of the experimental researches corroborate a correctness of the theses in the approach wordy formula to the given problem solution and ones allow to affirm an existence of direct and reverse connection of longitudinal (axial) and transversal (hoop and radial) stresses owing to an existence of orthogonal assemblage of the field lines (the principal stress lines) as a base of the field of internal forces in cylindrical rod. This base exists virtually in absence of external influence; this base exists really in all stages of elastic and plastic tension; this base exists in cylindrical rod also under action of lateral hydrostatic pressure only. In contrast to the explanation [14], in which the necking is bound with imperfection of the rod initial cylindrical form, the solution, given in this article, supposes geometrically ideal cylindrical form of the rod.

As regards the direct and reverse connection of transversal (hoop) and radial stresses with axial stress, the above mentioned results of Davis' [11] with the short thin-walled tubes of mild steel under action of combination the axial tension force and internal hydrostatic pressure testify to the following: the residual longitudinal deformation of tubular specimen is correspondingly decreased by 32, 39 and 43 %, when the designed transversal stresses under action of internal pressure were some lesser, equal and greater of the designed axial stress. If in Bridgman's experiments external (lateral) pressure was a cause of axial tension of the solid cylindrical rod up to its rupture, in Davis' experiments internal pressure has become an obstacle in the lengthening of tubular specimen under action of the tension force. Naturally, the steel tube cannot be sufficiently shortened under action of internal pressure only because of relatively small quantity of Poisson's ratio of steel. In this case, the rubber tube is the best example owing to a quantity ~0.5 of Poisson's ratio.

Thus, in the frames of the approach, stated in the given article, with the taking into account of the cited experimental results, the above mentioned well-known condition of a constancy of axial stress in cross-section of the heavy-walled cylinder under action of hydrostatic pressure should be described so:

- in the case of the internal pressure action

$$\sigma_t + (-\sigma_r) = -\sigma_z, \tag{5}$$

- in the case of the external pressure action

$$-\sigma_t = (-\sigma_r) = \sigma_z. \tag{6}$$

As applied to the tensioned cylindrical rod, the expressions (5, 6) are same correct for both its shell and core parts and in a whole. The expressions (5, 6) complete the solution of the Lame problem and ones ensure solution of the dragged out paradoxical situation, bound with a buckling of a long tube under action of internal hydrostatic pressure, created by pistons or by static head of a fluid flowing through the tube.

J.G. Panovko and I.I. Gubanova [14] declare that such "tube loses the static longitudinal stability, not experiencing the axial compression force in general."

Feodosjev [13] writes on "a false notion that in the stability question by Euler the internal compression force plays the basic role" and he adds "in reality it is not so." At the same time, these authors adduce the right solution by Euler, applying formally the axial compression force.

It is clear, that statements are based on the numerous experimental results, but not clear theoretical notion of the given phenomenon. Ascertainment of connection of the transversal forces with the longitudinal force and solution of the problem on internal forces, acting in the cylindrical rod under action of an axial tension force, adduced in the given article, clearly present a mechanics of origin of the axial internal compression force in a tube under action of internal pressure both under static conditions and a fluid flow through the tube.
Just the experiments, considered by authors [13, 14], are evidence of the direct and reverse connection of transversal and longitudinal internal forces in these cases.
The modern approach sufficiently corresponds to J.C. Maxwell conception (1856): "when (the forming energy) reached a certain limit, then the element would be fractured" and to M.T. Huber (1904) and A. Foppl and L. Foppl (1924) approach, cited by Timoshenko in his textbook [10]. Timoshenko adduces the example of a presentation of the rod stress state under action of a tension force in the kind of a sum of three-dimensional tension and pure (simple) shear. The modern approach, stated in this article, presents the stress and strain state field in its concrete form – with taking into account if the body form, of its material and of the kind of the load, applied to it.
The results of solution, adduced in the given article, can be applied to a number of sufficiently far spheres from it.
In particular, the soap water solution film, tensioned between two wire rings, accepts just the one-hollowed hyperboloid form, ensuring straightforwardness of the surface tension forces in absence of constructive rigidity of the soap solution. Those who bind a form of such film with a catenoid [15], fail to keep a principle: speak what you know. A drawing of gravitation is misplaced in this case. From two possible surfaces of rotation – cylindrical and hyperboloidal – the soap solution film, controlled by the intermolecular interaction energy, selects the latter on a principle of the energy minimum for creation of the form and correspondingly of a minimum of the surface area.
Other example, bound also with a liquid, is a breaking up of the free falling water jet, outflowing out of orifice (tube), into drops [15]. In this case, an increase of the jet fall velocity under action of gravitation causes the longitudinal tension forces in the jet surface and, as a consequence, an increase of the transversal tension force. In result of it, the jet is narrowed similar to the corset of the tensioned steel rod. The smooth narrowing of the jet increases spontaneously the transversal tension forces, leading to the necking in the kind of one-hollowed hyperboloid. Then a combined action of the longitudinal and transversal tension forces results to a narrowing of the neck in a point and to a breaking up of the jet. The breaking up is going on very quickly and one is accompanied by a sharp rebuilding of the hyperboloid form into two hemispheres. This transient process, in one's turn, is accompanied by a strong wave impulse, acting onto the jet upper and lower parts. Under action of this impulse, the longitudinal and transversal forces of the surface tension tear almost instantly the jet lower part onto drops. Thus the described very upper break of the water jet is a generator of its breaking up into drops.
Next example, bound also with the hyperboloid form, is a form of the bells: from the great at church belfry to the ship bell, hand bell. Sir Rayleigh had written in his fundamental work [16]: "as regards a form, accepted for the church bells, one is not received the satisfactory explanation till now." The Great Soviet Encyclopedia [17] contains the following thoughtful description: "the bell has a form of the hollow pear without its lower part." Pedoe notices in the above mentioned his book [2] that the neck cross-section of the one-hollowed hyperboloid, constructed by means of two wire rings and threads between its, has the ellipse form. The author of the given article has availed oneself of a diversity of the modern forms of polyethylene bottles and has purchased a bottle, containing the corset with 7.6 cm neck diameter and 9 cm diameter of its bottoms and 21 cm distance between its bottoms. Both bottoms of this bottle have the kind similar to a sphere and ones have sufficient rigidity of its form. At the same time a hoop rigidity of its corset turned out by extremely low. The slightest drawing out of air out of the bottle transforms right away a round cross-section of the corset neck into an ellipse. The slightest supercharging of the bottle

transforms the ellipse into a circle. The corset bases (the bottle bottoms) in both these cases keep a round form of its cross-section. A tapping by a finger-tip along the corset from one bottom to the other has allowed detecting on both sides from the corset neck a singly belt, where the tapping not causes characteristic sounding. The tapping outside these silence belts in both the corset neck zone and the corset bases causes characteristic low-frequency sounding with its attenuation during approximately 7 seconds. These silence belts is located in a junction of the hyperboloid neck concave profile with the convex profile beyond the neck. The same tapping on surface of cylindrical part of the bottle not causes the pure tone, and one lasts no more 1.5 – 2 seconds. In the next experiment with the corset bottle, the tapping was accompanied by a supercharging of the bottle. This experiment has shown the strong direct connection of the sound frequency with the supercharge pressure at the same duration of the sound attenuation. Then the corset bottle was shut off by the threaded lid and placed in the warm air stream. Under this condition the tapping has allowed ascertaining that the heating in the limits from $\sim 20^0$ up to $\sim 40^0 C$ leads to a rising of the sound tone practically by an octave. Observation of the corset surface and a listening of it in a process of the sound oscillations have allowed ascertaining that just the corset neck zone is a base source of the sound radiation. For all this, the corset neck cross-section accepts an ellipse form with a changing of its major and minor axes at every cycle of the oscillation and, correspondingly to it, the directional diagram of the sound radiation has four petals, and one of them coincides with a place of the tapping. So, the basic constructive features of the bell can be presented in the kind of two versions.

Accordingly to the first of them, these features are the followings:

- a form of middle surface of the bell lateral shell is the one-hollowed hyperboloid part, containing the neck part and adjoining to it from below the diverging part;
- an upper part of the bell lateral shell is restricted by a roof in the kind of a sphere segment with the tangent angle $\sim 35^0$ to its base plane; the roof contains in its center the bell hanger brackets;
- a lower part of the bell lateral shell is broadened to a diameter equal 0.75 – 0.8 of a height of the bell shell and one has along the lower edge a thickening – the percussive belt;
- in the whole the bell lateral shell has approximately the same relative thickness.

In this case the bell shell not contains the silence belt.

Accordingly to the second version, the lower part of the bell lateral shell presents by itself the convexo-convex evolventoid shell, forming the silence belt along its junction with the hyperboloid shell, which, in one's turn, is the concavo-convex shell.

Fig. 4 shows diagrammatically the mentioned version of the bell structure.

Owing to the silence belt, placed between the bell neck and the bell lower part, these two parts of the bell lateral shell oscillate in opposite phases. In both described versions a frequency of the sound oscillations, radiated by the bell, is inversely proportional to its neck diameter. The described structure of the bell form allows giving concrete recommendations for assuring the radiated sound tone, but it oversteps the frames of the given article.

Detection of the silence belts allows explaining why the initial transversal crack in a core of the tensioned cylindrical specimen, as stated above, arises just in the plane, passing through a junction of the hyperboloid and evolventoid surfaces. The fact is that, the deformation and plastic flow of the specimen are accompanied by longitudinal and transversal oscillations of a sufficiently high magnitude. A change of a curvature of the longitudinal profile from convex in the evolventoid zone to concave in the hyperboloid zone leads to that the transversal (radial) oscillations are going on in opposite phases in these zones. In a result the above plate is a plate of action of the radial cyclic shear stress, promoting a shaking of the specimen material structure. In this case the above silence belt renders negative influence onto the specimen strength.

Reverting to the base object of the given article, it is also necessary to note, that the necking is a typical phenomenon for such metals as mild steel. At the same time the features of the metal internal structure lead not only to different modifications of the necking, but to a fracture without the necking. For example, the cylindrical specimen of grey iron under action of an axial tension force is fractured without visible residual deformation (the square-break fracture. Such

distinction of the specimen behavior is explained by that, the ultimate strength of grey iron is in 2 – 5 times less than its compression strength. Therefore, in contrast to the steel specimen, the fracture of the grey iron specimen is going on by means of a forming of transversal (hoop) crack on its external surface with its subsequent fast development to the specimen axis.
It is quite probably, with the taking into account of the above mentioned Bridgman experiments, that the grey iron specimen will be fractured with the necking under action of combination of sufficiently high hydrostatic pressure and comparatively not great axial compressive force.
Next question, affected in the end of the Solution section of the given article, on pure uniaxial stressed state, has the great methodological significance and one requires the more detailed explanation. The fact is that, the traditional course of strength of materials from the very outset presents the rod tension as a “simple tension.” Determination of a stressed state in it is restricted by division of the tension force by the rod cross-section area. This simple tension is then presented as uniaxial stressed state. Further the traditional approach envisages a study of planar stressed state and then three-dimensional (3-D) stressed state as complex stressed state. In that way, traditional teaching is formally constructed rightly: from the simple to the complex. At the same time the results of the tension tests of thin wires of mild metals – with wire diameter, compared with the grain diameter of the wire metals – show that in these cases the fracture is going on without the necking (see, for example, I.J. Dechtjar’s article in the above-mentioned “Collected articles” [3]). This feature in comparison with the results of the tension tests of the standard specimens of mild steel testifies to that the rod with its diameter 10 – 20 mm is presented in the kind of a bunch of thin wires with its total cross-section area equal the solid rod cross-section area. In a result, those, who had good studied the course of strength of materials, are astonished by the necking, and those, who had weakly studied this course, do not think about it. This paradoxical situation exists more than 100 years.
In this connection, it is appropriate to appeal to prof. L.R.G. Treloar book [18], in which the author adduces an example of tension of the usual polyethylene film strip 10 cm long and 1 cm width. In this experiment, the tension is accompanied by shoulder effect in the kind of the local narrowing of the strip, which envelops gradually all its tension part similar to the local narrowing, enveloping all length of the mild steel rod, tensioned in initial stage of its plastic deformation. But, in contrast to the steel rod, the polyethylene strip fracture is going on without the necking, i.e. practically brittle. Such fracture of the very plastic material is stipulated by the following. In initial stage of plastic deformation, the polyethylene molecular chains straighten oneself and ones are stretched along the tension direction, losing a part of its transversal ties. In a result of it, the entire material is transformed practically into a bunch of microfibers, the every of which and ones together are fractured without the necking. Such entire material is more similar to the wire rope than the solid rod.
This is what, in brief, follows from solution of a problem on the “simple tension” and on the necking.
In conclusion of the discussion it is necessary to note the successive solution of the stated problem is the next in turn example of using of Physical Ensemble method, developed by author of the given article. Effectiveness of the method for physically adequate and mathematically sufficiently strict construction of the models of some phenomena in a field of the fluid dynamics was showed in previous articles [19 - 28] of the author.

Final remarks

The approach, the solution of the problem, and the considered examples, stated in the given article, create necessary prerequisites for fruitful solution of the problems and questions, bound with an analysis of the strained and stressed state of the solid and fluid on the new united physically sensible methodological base.

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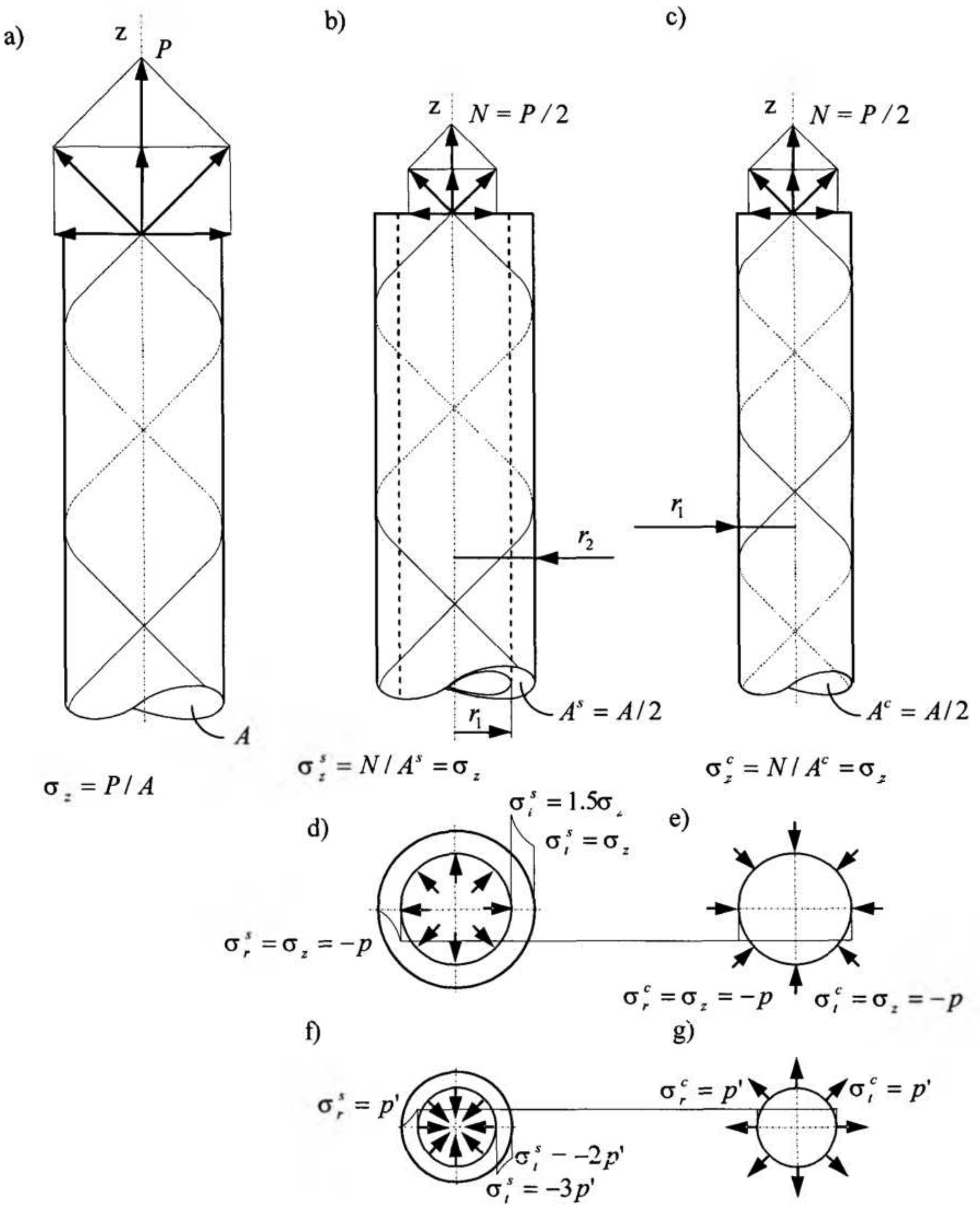

Fig. 1. Diagrams of the forces and stresses for the cylindrical rod under action of the axial tension force: a, b, c) diagrams of the forces and the field lines, acting in the rod and its parts - shell (superscript ***s***) and core (superscript ***c***) - correspondingly; d, e) diagrams of the transversal - hoop and radial - stresses in the rod shell and in the rod core, correspondingly, and its mutual pressure in the initial stage of the rod plastic deformation; f, g) diagrams of the transversal - hoop and radial - stresses in the rod shell and in the rod core, correspondingly, and its mutual pressure in the neck zone

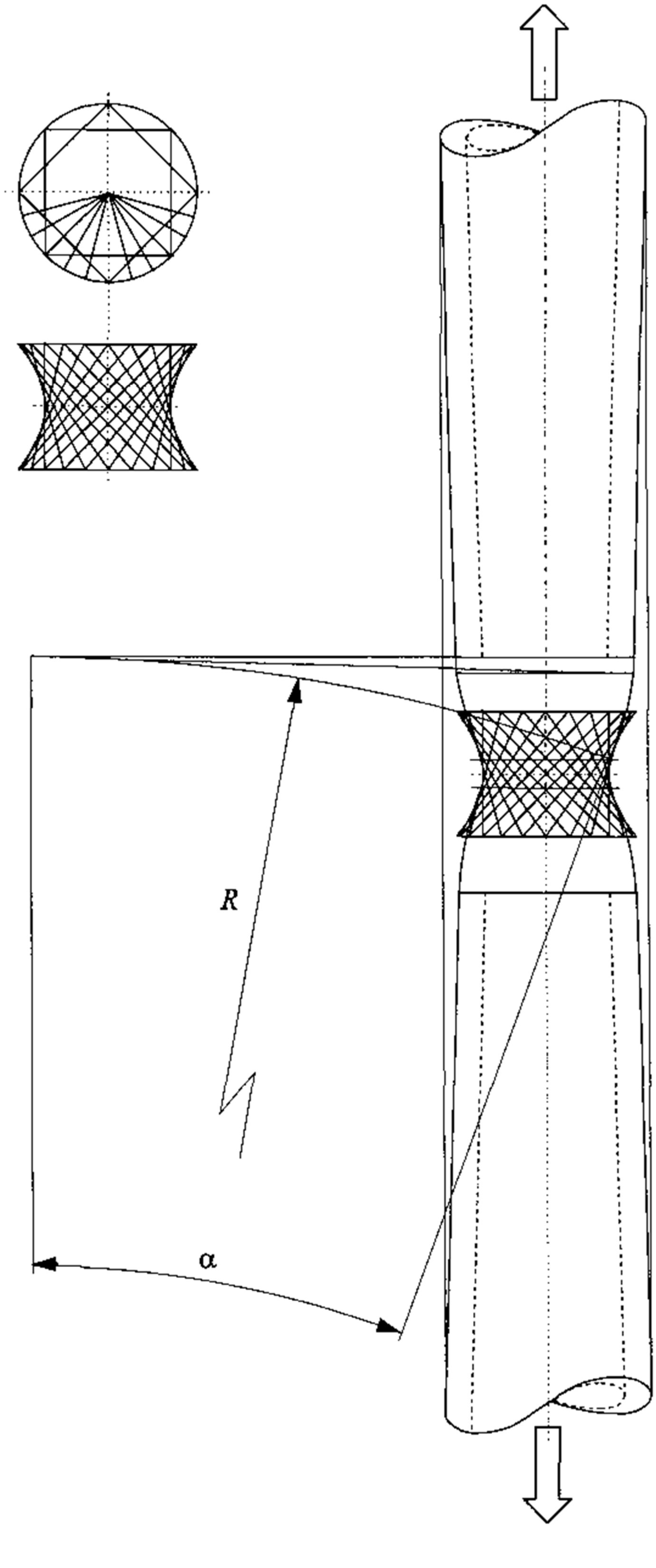

Fig. 2. Diagrams of a constructing of the one-hollowed hyperboloid and of the longitudinal profile of the tensioned cylindrical rod on the necking stage under action of the axial tension force only

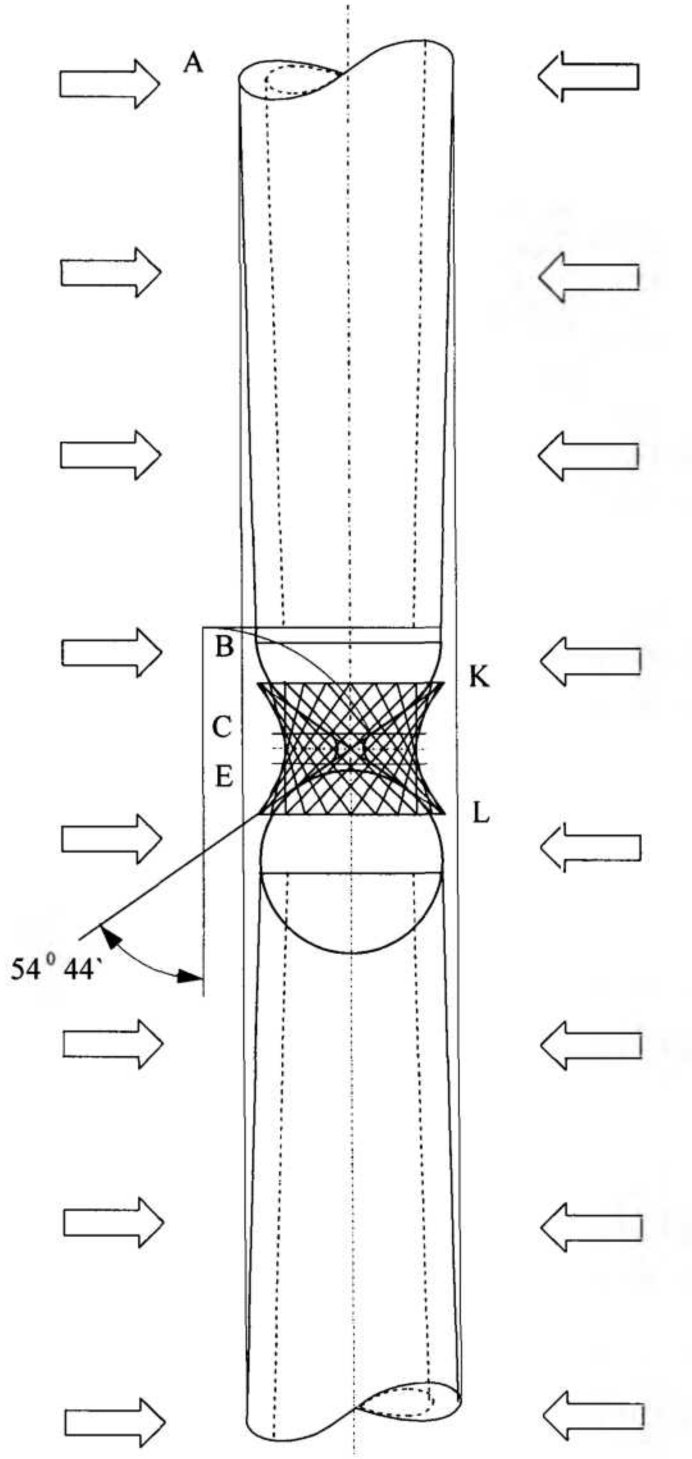

Fig. 3. Diagrams of a constructing of the longitudinal profile of the tensioned cylindrical rod on the necking stage under action only of the lateral hydrostatic pressure: the development angle of the evolvent reaches the maximum quantity 54^0 44'; AB - the hyperboloidal corset; BC - the evolventiod belt; CE - the orthogonal hyperboloid belt; KL - the orthogonal hyperboloid in the hyperboloidal corset

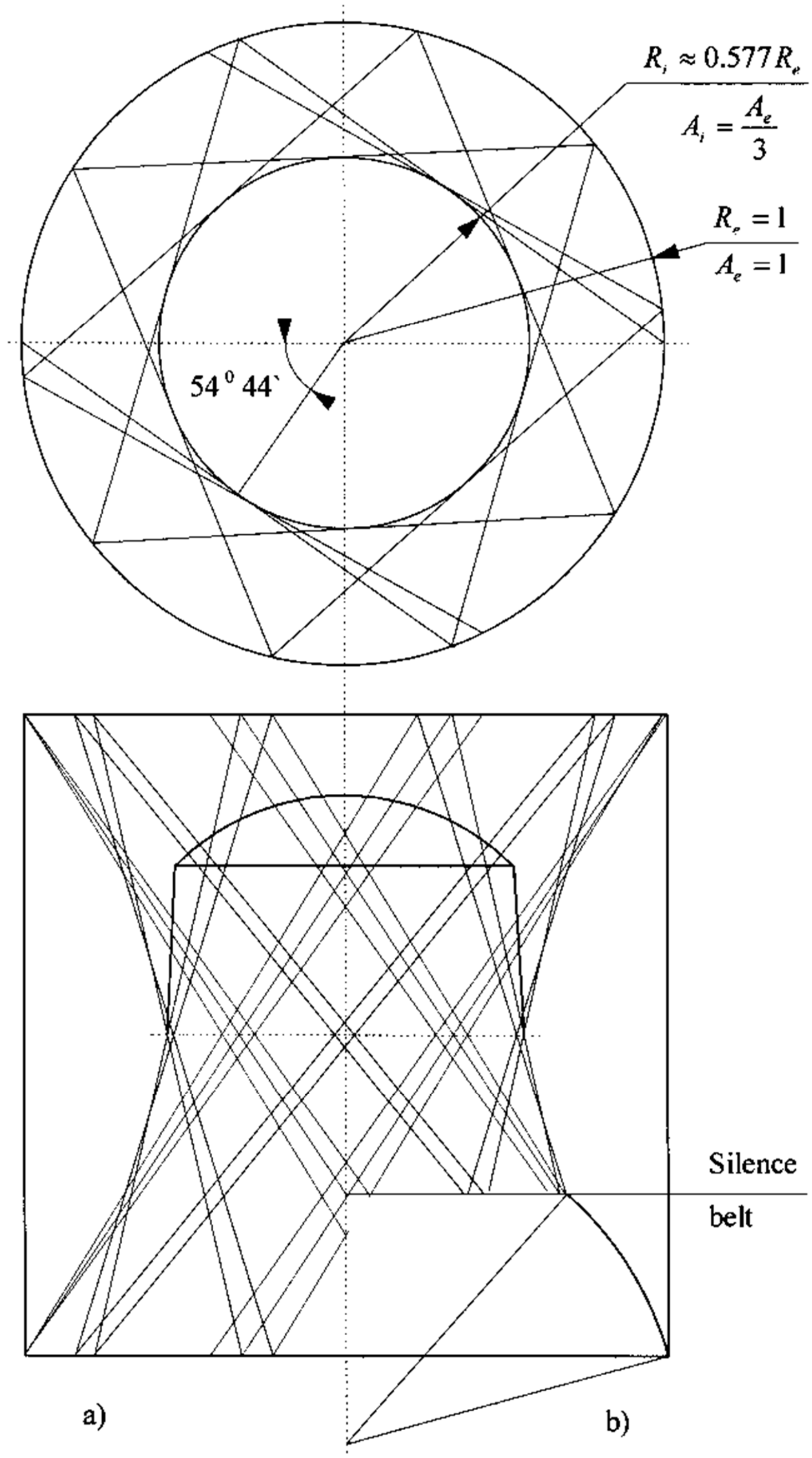

Fig. 4. The construction diagram of the middle surface profile of the bell (the Tsar-bell in the Moscow Kremlin): a) combination of hyperboloid and cone; b) combination of hyperboloid, cone and torus (evolventoid)