Mechanics of solid body, hydromechanics and gas dynamics in conic sections as a way for solution of the problems

Part 1: The conics in axisymmetric solids and constructions under simple loads

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It is offered the results of graphic-analytic solution of problems on change of a form in cylindrical, prismatic and flat bar from elastic-plastic material immediately before its fracture under tensioning, compressing and twisting. It is adduced examples of rational constructions for the high pressure vessel and others.
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Introduction

The given article is a sequel and development of contents in the author previous articles [1, 2].
A subject of study in the given article is a problem on change of a form of axisymmetric specimens from statistically isotropic material under axisymmetric load, increased uniformly up to the specimen fracture.
History of this question is bound up in the main with wide using steel for the machine elements and constructions. In XIX century it has been created machines for experimental estimation of strength of the steel specimens in kind of cylindrical and prismatic bars under longitudinal tension up to its fracture. These experiments for the first time showed characteristic feature in behavior of the tested specimens from medium- and low-carbon steel:
- in the beginning of loading: elastic (reversible) lengthening of very not great quantity;
- then, as a tensioning force is increased: it is going on not great plastic (irreversible) lengthening, accompanied by conversion of cylindrical form to smooth, hardly visible, contraction from the specimen ends to its middle; its shining surface becomes mat;
- at subsequent uniform increase of loading: it is suddenly arising large local contraction of the specimen near its middle, here it is occurring rupture, accompanying by bang;
- temperature of the rupture surfaces is increased from room to some more ~100°C;
- the rupture surface contains two parts – central and bordering; the central part is oriented across to the specimen and one has a crystal structure; surface of the bordering part is inclined under ~35° to the specimen axis, and one has longitudinal tracks of sliding.

In contrast to it, initial form of cylindrical specimen is remained in the whole under torsion moment up to its fracture, although its cross-section is some decreased and its length is some increased. In his capital book [3] A. Nadai named the lengthening by J.H. Pointing's effect by name of English scientist who had for the first time detected the phenomenon (1912). Fracture of the specimens was going on in kind of transversal circular crack with its beginning from the specimen surface.
The-state-of-the-art of the problems: in previous articles [1, 2] of the author it has been given descriptive solution abovementioned problems.
The present article completes solution of these two problems by means of graphic-analytic method and generalizes the method on examples of axisymmetric bodies and constructions.

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Approach
Principal physic conceptions about kinematics of change of a form and corresponding to it 3-dimensional stressed states of axisymmetric bodies from statistically isotropic material (in particular, plastic metals) are stated in above-mentioned articles of the author.

Solutions
Fig.1 shows reciprocal geometric connection of curves from conic sections conformably to axisymmetric bodies in kind: right circular cylinder with its radius, R, the right one-hollowed hyperboloid of rotation inscribed in it, paraboloid of rotation inscribed into the hyperboloid, and evolventoid of rotation conjugated both the cylinder and hyperboloid. Unity of origin of these curves (from right circular cone) ensures constructing of diverse forms of axisymmetric bodies by means of its combination. This is so called graphic formula of geometric compatibility (GFGC) of such curves and surfaces for bodies of rotation mainly.

Fig.2, in its upper diagram, shows geometric interpretation of fracture of cylindrical specimen from very plastic material – aluminum, heated up to 91% of its melting-point, which one is adduced in fig. 69 of above-mentioned book [3]. The diagram is constructed by means of two pair of evolventoids of rotation. Lower diagram shows one of stages of plastic deformation, foregoing the specimen fracture, in kind of combination of a pair of evolventoids of rotation (made from evolvent of circle) and conjugated with them the right one-hollowed hyperboloid of rotation, forming so called neck. (Generatrices of the hyperboloid are straight lines which ones are inclined at ±45° to the specimen axis).

Fig.3, in its upper diagram, shows a simplified diagram of constructing of the neck in cylindrical specimen under axial tensioning. Basic element for constructing of the diagram is the right one-hollowed hyperboloid of rotation with diameter of its bases equal to the specimen diameter. Arch of the circumference evolvent in range from 160° to 180° forms the evolventoid surface profile, which one unites smoothly profiles of the specimen cylindrical part and the hyperboloid in the neck zone. Such uninvolved construction reproduces a form and proportions of the specimen external surface and allows reproducing a form and proportions of the boundary surface between the shell and core parts of the specimen in the following succession of actions:
- make a copy of already constructed hyperboloid and decrease it proportionally to 70.7%;
- place the decreased hyperboloid in the neck center;
- make a copy of already constructed evolvent of circumference and increase it proportionally so that diameter of its circumference becomes equal to the specimen neck length;
- restrict arch of the evolvent in the limits from 164° to 180°;
- place the arch so that to ensure its smooth join with the specimen core cylinder and with profile of the restricted hyperboloid.

In a result of the graphical constructions, showed in Figs 2, 3, we obtain geometrically strict structure of neck for cylindrical specimen of sufficiently plastic material under axial tension. This result is evidence to that principal stresses – own and arising under tensioning load – in the cylindrical bar shell act along the trajectories of two orthogonal sets of helical lines and ones retain its orthogonality on all stages of plastic deformation. Just the principal stresses determine change of a bar form from the straight circular cylinder to the right one-hollowed hyperboloid of rotation retaining its inclination ±45° and at the same time transforming helical curves into straight lines. In process of such local change of the bar form, a pair of evolventoids of rotation ensures smooth geometric compatibility of circular cylinder and the hyperboloid. In other words: in contrast to longitudinal straight lines, as generatrices of geometrical – abstract – form of straight circular cylinder, trajectories of principal stresses inclined at ±45° are generatrices of the right one-hollowed hyperboloid of rotation in physical – material – cylindrical specimen.
Here it is showed also boundary between shell and core parts of the specimen, constructed by the same way. Besides that, here it is showed, in red lines, a place of a primary tear origin – in

central part of near-axial singular rod cross-section and trajectory of its radial development up to the specimen shell part which one is fractured by shear at angle close to 55°.
It should be supposed that above-mentioned internal structure, including in itself: shell, core, near-axial singular rod and also two equal parts of cylindrical specimen length, exists in it not only under action of axial load but also in lack of load, in its free state. At the same time, a limit of proportionality, at testing by tension, is measure of internal energy of the specimen material keeping the specimen as solid body with its initial isotropic material in lack of the load. On stage of yielding, the specimen elasto-plastic isotropic material transforms into orthotropic owing to lining up of some part of structural imperfections along trajectories of principle stresses (in kind of two sets of helical lines). Now it starts elasto-plastic interaction of shell and core in which the shell is a leading factor: in consequence of helical trajectories of principle stresses, the shell is tensioned in axial direction and at the same time one presses out the specimen core promoting plastic lengthening of the latter up to necking and fracture of the specimen.
In contrast to it, in axial pressing, these shell and core switch the roles: now core is elasto-plastically increasing its diameter and thereby one stretches shell up to fracture of the latter; note should be taken: correlation of diameters of the shell and core remains constant and equal to its initial quantity 0.707 up to the specimen fracture.

The same Fig.3, in its lower diagrams, shows flat bar from plastic material (medium-carbon steel) under longitudinal tension and rigid restriction of transversal displacement of the test machine claws. Numerous experiments shoved that fracture of such specimens is always accompanied by origin of linear neck in kind of two-sided thinning at ~ 55° to longitudinal axis of the bar and at ~ 35° to its cross-section plane. These two directions are orthogonal each other and they are turned at 10° relatively to set of principal stresses, inclined at $\pm 45°$ to the bar axis. At the same time the incline angle of the linear neck coincides with that for a diagonal lying in diametral section of the right one-hollowed hyperboloid of rotation.

Fig.4 shows simplified diagram of constructing of neck in specimen in kind of right prism with rectangular cross-section under axial tensioning. Such prismatic body, in which dimensions of its cross-section are commensurable, i.e. B/H < 7, one, just as cylindrical body, has structure of its stressed-deformed state in kind of shell and core. Ratio of dimensions in the core cross-section is the same as that of the body cross-section, but its quantity is decreased by factor $1/\sqrt{2}$, as that is usual for cylindrical body. Basic element for constructing of the diagram is the right one-hollowed hyperboloid of rotation just as in the case of cylindrical body. Feature is that in this case it is necessary to exploit two sizes of the hyperboloid according to width and thickness of the specimen and, of course, two evolvents, corresponding to that. The neck cross-section form is quite similar to the body cross-section. In the given example it is shown constructing of neck only for the specimen width, in flatness ZX. In order to construct of similar diagram for the specimen thickness, in flatness ZY, it is sufficient to make a copy of the neck in flatness ZX (it is outlined by blue frame) and then to decrease its width only by means of scale $(H/B)100\%$ – since the neck length, L, is the same on all sides of such specimen. A type and succession of the specimen fracture also quite corresponds to that of the above-described cylindrical specimen: in beginning, initial brittle crack arises in central zone of the cross-section core, then the crack dimensions is increased up to the core - shell bound, after that the specimen shell is fractured by shear.

Fig.5, in its left side, shows schematically cylindrical bar under action of two pair of forces, forming the torsion moment. The bar length is restricted by one coil (pitch) of trajectories of principal stresses under 45°. Development of the trajectories in kind of diagonals (red line emphasizes tension, blue line – compression) is superposed onto development of the bar cylindrical surface. Numerous experiments show that fracture of such bars of plastic metals under the torsion moment is going on in kind of transversal circular shear while fracture of such bars from brittle metals is beginning by crack along the helical line, under ~45° to the bar axis, in one side of the bar cylindrical surface and one is completed by break along the cylinder longitudinal generatrix on opposite side of the cylindrical surface. The fracture length along the bar axis is approximately 1.5 of its diameter. In both cases the specimen fracture is not accompanied by visible change of its form.

In order to offer general rational explanation to such results of experiments, it is sufficiently to exploit the previously stated in [1, 2] notion of the author that principal stresses are not only those normal stresses, along which the shear stresses is in lack. Conformably to solid body from statistically isotropic material, the principal stresses are evidence to presence in it of own, initial field of forces. Just the field:

- retains a body form in a lack of external loads;
- ensures reversibility of change of a body form under not great external loads;
- determines change of the body form on stage of its plastic deformation;
- determines a solid body strength and a kind of its fracture.

The field is the same both for plastic and brittle material. Although specimen from brittle material is fractured without visible change of its form because of comparatively low tensile strength of the material. In contrast to it, an example of tensioning of cylindrical specimen, in Fig.3, shows that own field of forces changes its form, but incline of the force lines retains significantly invariable. The other factor, determining the stress state structure of a solid body, is bound up with micro-scale high-frequency oscillations of micro-particles constituting the body. These fundamental oscillations provoke macro-scale low-frequency oscillations in conformity with the body form and dimensions, and, as the result, create macro-scale substructure in the body isotropic material. Diagrams in Figs.3, 4 show not only possibility but also necessity of existence of a core and enveloping it shell in compact isotropic volume of axisymmetric body. Areas of cross-section of the core and shell are equal between itself and ones divide a tension force, applied to the body, on two equal parts. Principal stresses, as reflection of the force field lines in the body volume, act along two reciprocally orthogonal sets of helices under $\pm 45°$ to the body axis. Metrical pitch of the helices is $t_m = \pi \mathrm{D} \cdot tan 45° = \pi \mathrm{D}$, where D is the specimen diameter, and $t_{nd} = t_m/\mathrm{D} = \pi$ is its relative pitch as a dimensionless quantity.

Relative, dimensionless, quantity of the right cylindrical bar length can be also expressed in its diameters in the kind $\overline{\mathrm{L}} = \mathrm{L}/\mathrm{D}$, used in hydraulic calculations.

At the same time metrical length $l/2$, expressed by relative pitches of helices, $[l/(2\pi)],^2$ is contained in Euler's formula, determining the buckling force quantity. L. Euler had showed (1744), that the bar axis curvature under longitudinal bending follows plane sinusoid in the limits from 0 to π. Now we turn our attention to a well-known fact, that the sinusoid is visible profile of helix on surface of transparent circular cylinder. An incline of trajectories of principal stresses to the bar axis stipulates radial pressure of the bar shell onto its core under action of tensile load, applied to the bar.

Only now, using above-stated physically adequate notion, we can hope to right solve of a problem on the stressed and deformed state of cylindrical specimen under action of a torsion moment on all stages of its loading up to its fracture.

Thus, specimen in kind of right circular cylinder with its diameter, D, and its metrical pitch length, $t = \pi \mathrm{D}$, is loaded by a torsion moment, M, in kind of two pair forces, F, applied along the tangent to a contour of its ends, i.e. $\mathrm{M} = 2(\mathrm{F} \cdot \mathrm{D}/2) = \mathrm{F} \cdot \mathrm{D}$. These forces load one set of helical lines in own initial field of force of the specimen with additional tension and the same time ones loosen tension of the other set of helical lines, orthogonal to the first, up to its compression. The additional tension in its turn provokes longitudinal tension and radial compression of the specimen, owing to an incline 45° of the force helical lines. Linear force of radial compression is $p_l = 2 \cdot 2\,\mathrm{F}/\mathrm{D} = 4\,\mathrm{F}/\mathrm{D}$, and specific radial pressure is respectively $p = p_l/t = 4\mathrm{F}/\pi\mathrm{D}^2$. Quantity of the pressure on the specimen cylindrical surface is apparently equal zero, but one accumulates successively and increases to its axis. Character of the increase is determined by means of the known expression for polar moment of inertia of annular layer. In the result we

obtain $p\ (r) = 4\mathrm{F}\,[1-(r/\mathrm{R})^4]/(\pi\mathrm{D}^2)$ in the limits $\mathrm{R} \geq r \geq 0$, i.e. radial pressure is increased according to parabola in the fourth power from zero on the specimen external surface to maximum quantity $p_{\mathrm{r}=0} = 4\mathrm{F}/(\pi\mathrm{D}^2)$ at its axis.
Combination of radial and together with its longitudinal compression in near-axis zone of the specimen with longitudinal tension in its near-surface layer is evidence of existence of macro-scale substructure in kind of core – shell in the specimen volume. Condition for existence of the substructure is in equality of polar moments of inertia of solid cylindrical bar and tube as its analogues, i.e. $J_{pc} = J_{ps}$, where $J_{pc} = \pi\mathrm{D}^4/32;\ J_{ps} = \pi\mathrm{D}^4[1-(d/\mathrm{D})^4]/32$, from where $d^4 = \mathrm{D}^4[1-(d/\mathrm{D})^4]$ we obtain $d = \mathrm{D}\ /\ \sqrt[4]{2} \cong 0.841\mathrm{D}$. Fig.5, in its lower right side, shows the radial pressure profile under twisting of the specimen and, in its upper right side, – a profile of longitudinal, axial stresses in the specimen cross-section. Graph of parabola in the fourth power is shown in its comparison with parabola in the second power in lower left side.
Condition of the specimen fracture from plastic metal is that to the above-determined axial tensile stresses in the specimen shell it is necessary to add axial stress in it, conditioned by axial component $\mathrm{F_z} = 2\mathrm{F}sin^2 45° = \mathrm{F}$, in the tensioned set of the force lines, and then we obtain $\sigma_{\mathrm{zs}}^{\mathrm{F}} = 4\mathrm{F}/[\pi\mathrm{D}^2(1-0.841^2)] = 4.35\mathrm{F}/\mathrm{D}^2$. The two-dimensional stressed state of tension near the shell external surface in kind of a sum of the axial tensile stresses and annular tensile stresses determines origin of initial annular crack as a beginning of the specimen fracture without visible change of its initial form. Solution of the problem on connection of axial and radial stresses is adduced in [4] as applied to a concrete column placed in tube and loaded with axial force.

Fig.6, in its upper part, shows profiles of the axial compression stresses (dark blue lines) and tension (red lines) in zone of short change of a shaft diameter under torsion moment in lack of bending load. Superposition of the longitudinal compression profiles in the core of the shaft parts leads to an increase of the longitudinal tensile stresses near external surface of the shaft shell with its lesser diameter in zone of short change of the shaft diameter.
Diagram in center of Fig.6 shows possibility of geometrically strict division of a circle area with its quantity equal to unity onto circles with divisible to its area. The division is in the main made by means of successive inscribing of right hexagon, square, and equilateral triangle and one visually testifies to a leading part of cylindrical core of such prismatic bodies in its resistance to a torsion moment action. Its cylindrical core remains initial form and the projecting parts of these prisms as if wind up along the helical trajectory around core under twisting. Of course, the cylindrical core of these prisms contains its own core and shell parts. In the whole, the obtained diagram turned out, suddenly for the author, very much like ordinary hexagonal nut.
Diagram in lower part of Fig.6 contains – in black lines – treated by St. Venant (1854) a cross-section of elliptic cylinder, turned under action of the torsion moment in direction of arrows, which one adopted from S.P. Timoshenko book [5]. (Similar diagrams treated by St. Venant for prisms with its square and equilaterally triangular cross-section are adduced by F.H. Newman and V.H.L. Searle in their book [6]. Similar diagrams of transversal – helical - motion of liquid in tubes with its square and equilaterally triangular cross-section are adduced by L. Prandtl in his book [7]. A cause of such motion of liquid in above-mentioned tubes is explained by author of the given article in one of his previous articles [8]). A core of the elliptical cylinder is shown by the author in kind of right circular cylinder with its own core in red lines. Interaction of the elliptical cylinder parts, projected beyond the bounds of its circular core, with the latter under action of the torsion moment quite corresponds to above-described to prismatic bodies.

Fig.7, in its upper part, is the next graphical formula of geometric compatibility for flat curves: evolvent of circumference in the second quarter, equilateral hyperbola and surfaces formed with these curves. In the same place it is shown parabola in the second power, conjugated with the hyperbola in the limits of height of the right one-hollow hyperboloid of rotation.
The same Fig.7, in its lower part, shows the construction element in kind of a hollow body of rotation under axial compressing load. The body middle surface is the right one-hollow

hyperboloid of rotation, and a curvature of profile of its internal and external surfaces ensures equality of the body cross-section along the middle surface profile. Such form is optimum for the body which is made from plastic metal, for example, medium-carbon steel. At the same time such body, made from brittle material, for example, cast iron or ceramic must be smoothly thicken to its both butt-ends. A. Nadai in his book [3] adduced such body diagram for ceramic.

Fig.8 shows the superposed diagrams of the bottom profiles with central hatch and lid for the cylindrical high pressure vessel. In these diagrams a conjugation of the vessel cylindrical part with spherical part of its bottom with its doubled radius and also a profile of the lid concave shell are presented in kind of the circumference evolvent arch in the second quarter. In contrast to the well-known profiles of the bottom in kind of torus-spherical or torus-elliptical shells, the evolvent ensures a most smooth conjugation of the vessel cylindrical shell with its bottom spherical shell of the doubled radius owing to optimum change of a quantity of its curvature radius. The same property of evolvent creates possibility for decreasing of the force radial trust of the hatch lid and thereby one allows to a considerable extent increasing of the lid diameter; in ones turn, such possibility allows decreasing of the vessel length, remaining its capacity. At the same time, since the lid concave shell is in the compressed state under the vessel internal pressure, we can decrease its weight by a use of light materials with its quite quantity of a ratio of the elasticity modulus to specific gravity: from the aluminum alloys up to the laminated plastics, and also by a use of annular dowel joint. Although thickness of such lid can be found to be larger of that of the bottom shell, its weight and simplicity of its fastening ensure an advantage in comparison with convex lid. A zone of the calculated-designed search of the optimum construction for the bottom and lid is shown in blue lines, and red lines show a zone of the bottom profiles for vessels of the opened type used as reservoir for transporting of liquids.

Fig.9, in its upper part, offers an example of a use of the evolvent arch in the second quarter as a bottom profile and of the evolvent part in the limits of 135…180° as a profile of the hatch lid for constructing of the high pressure vessel diagram. A profile of circular joint of the lid with the bottom hatch contains two trenches: for sealing ring and set of dowels. Cylindrical part of the vessel together with its bottom can be manufactured from high-strength steel of equal thickness and the hatch lid – from the high-strength aluminum alloy. A set of dowels in kind of the ring sectors can be manufactured from material of the vessel basic elements. The lid concave shell must be calculated not only on compression strength but also on a bending stability and its contact surfaces with hatch – on bearing stresses. Feature of use of the circumference evolvent is that its initial proportions must be remained in any graphical constructing. In any case it is sufficiently to change a scale and the curve length. A photo, placed under the vessel fragment, shows concave lid from high-strength aluminum alloy (analogue of aluminum 7075-T6) after its hydraulic test, carried out for verification of the calculated results. Diameter of the lid is 800mm; thickness in its near-polar part 5.2 mm. A fastening of the lid in testing device for hydraulic test corresponded to diagram showed in upper part of fig.9. The pressure quantity of its buckling, accompanied with its local fracture, was found to be equal 87 $\mathrm{kgf/cm^2}$ as against the calculation range 85…92$\mathrm{kgf/cm^2}$. These results have been got by the author more than thirty years ago.

Fig.10, in its upper part, offers diagrams of constructing of the high pressure vessel bottoms, manufactured by winding of high-strength synthetic filament.
Below, in the same Fig.10, it is shown diagram of the cylindrical vessel with central hatch in its bottom where is placed circular flange carrying a lid with its concave shell and also a sealing ring and a set of dowels. A profile of internal surface of the bottom is arch of the circumference evolvent in the second and partially in the first quarter.

Fig.11, in its left part, offers short cylindrical specimen from plastic metal, which under axial compression takes, in the beginning, the shape of a barrel and then the shape of a tablet with

radial cracks in its circular contour. The barrel form is none other than combination of hyperboloid and evolventoid of rotation, shoved in Fig.3. Photos of three specimens after testing and the enlarged photo of the specimen, cut half-and-half, are all adopted from V.G. Osipov article in [9]. Author of the given article places two concentric white circumferences as a bound of the specimen initial cross-section R_i and also a bound between its core and shell $r_d = R_d/\sqrt{2}$ after plastic deformation. The core part of the specimen is found in a state of volumetric pressing, and one excites circular (tangential) tension in the shell part up to beginning of its radial cracks.
At the right below, **fig.11** offers photo of a disk surface subjected to high internal pressure. This result had been obtained by L. Hartmann (1896) and one is adopted from Nadai's book [3]. Evolvent of circumference cannot be competitive in comparison with logarithmic spiral for reproduction of Luders tracks in this case. Just the logarithmic spiral as equally-angled spiral with its development at an angle of 45° corresponds to helical lines on circular cylinder and rectilinear generatrices of the right one-hollowed hyperboloid of rotation showed in **figs 3, 5**.

Fig.12, in its upper part, shows thin-walled shell in kind of a right circular cylinder from elastic-plastic material. Orthogonal assemblage of longitudinal straight lines and circumferences offers the cylinder as abstract geometrical surface. At the same time, orthogonal assemblage of trajectories of initial principal stresses in kind of helical lines under 45° to the cylinder axis offers the cylinder as physical body. This thin-walled shell turns into ribbed cylindroid under impact axial compression. Such cylindroid surface is called Schwarz's (1880) surface. The photo has been obtained by A.P. Coppa (SSL, General Electric, 1960) and one is adopted from A.S. Volmir book [10]. The photo shows distinctly that formation of trigonal-rhomboidal depressions and convexities occurred by way of straightening of trajectories of principal stresses under condition of transversal restriction in the shell perimeter.
Fig.13 shows a right trihedral prism with its cross-section in kind of equilateral triangle as three-dimensional body with minimum quantity of its lateral sides. Rotation of the prism about its longitudinal axis leads to formation of cylinder by its ribs and hyperboloid - by diagonals of the prism sides – which one is inscribed to the cylinder. The same diagonals in kind of bars, joined in pairs between itself and with two bar equilateral triangles, form a rigid bar construction in kind of tripod, known in ancient times.
Fig.14 shows a thin-walled biconic shell of rotation from sufficiently durable material. Not great rumpling of upper part of the shell causes in it a pair of elastic waves. At the same time the same pair of elastic waves origins in lower part of the shell, but with right angle turn. Thus rotation of one, driving, pair of waves causes rotation other, driven, pair of waves without rotation of the shell itself. This is so called toothless wave gearing.

Discussion of results
The examples, produced in Figs 2, 3, 4, 5, 6, 11, corroborates rightness of physical conceptions for solution of problems on a change of a form of simple axisymmetric specimens under simple axial tension, compression and torsion immediately before and in the process of its fracture. These conceptions allow for the first time to solve the problems on the stressed and strained states under not only tension and compression but also under torsion using only principal stresses. The Luders lines, well-known from middle of XIX century, are a way out of streams of dislocations and vacancies out of a body depth onto its surface along the trajectories of principal stresses. In the result a plastic deformation is accompanied by the experimentally observed formation of the filament structure on a deformed body surface.
A use of the conic section curves is a basis for graphical solution of problems in the given article.

Final remarks
Author dedicates the given article to memory (445 years) of Johann Kepler, who had laid down the foundations of kinematics and for the first time had used the conic sections in mechanics.

Acknowledgements
Author expresses his deep gratitude to his son Alexey for help and supporting.

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[1] S.L. Arsenjev, http://uk.ArXiv.org/abs/physics/0609.49
[2] S.L. Arsenjev, http://uk.ArXiv.org/abs/physics/0809.4026
[3] A. Nadai, Theory of Flow and Fracture of Solids, vol.1, McGraw-Hill Book Co. Inc., Pittsburg, PA, 1949
[4] S.P. Timoshenko, Strength of material, Part I, 3rd Ed., D. Van Nostrand Co., Inc., Princeton, NJ, 1955: Chapter II, sect. 18, prob. 3 (A concrete column is placed into a steel tube)
[5] S.P. Timoshenko, History of Strength of Materials, NY, 1953
[6] F.H. Newman and V.H.L. Searle, The General Properties of Matter, 4th Ed., Edward Arnold &Co., London, 1950
[7] L. Prandtl, Fuhrer durch die Stromungslehre, 2nd Ed., Gottingen, 1944
[8] S.L. Arsenjev, http:// http://uk.ArXiv.org/abs/physics/0711.1705
[9] V.G. Osipov, A process of fracture by shear under simple compressing and tensioning / in: Strength of Metals, Academy of Science Publishing, Moscow, 1956
[10] A.S. Volmir, Stability of Elastic Systems, State Publishing of Physic-Mathematic Literature, Moscow, 1963

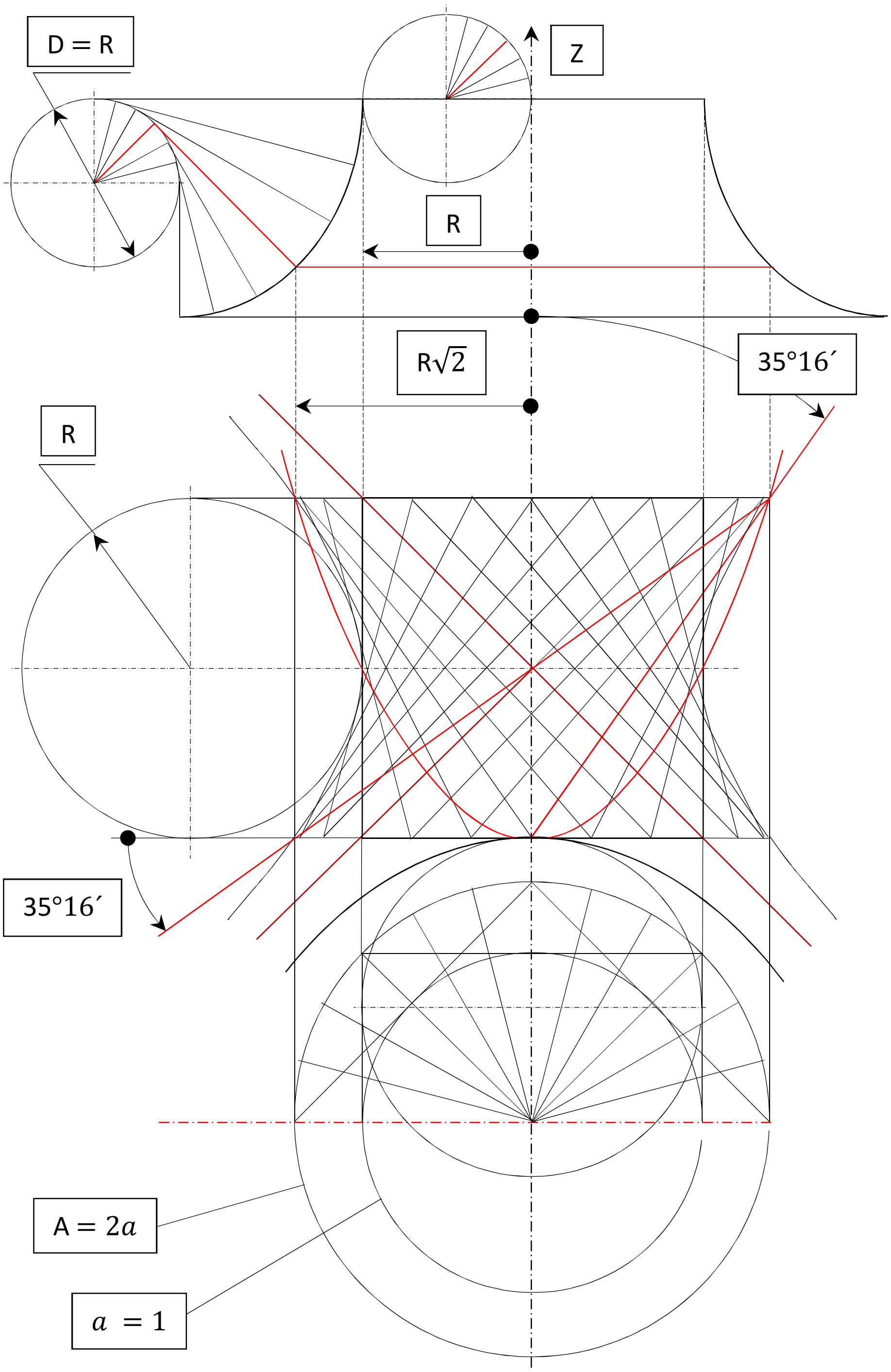


Fig.1

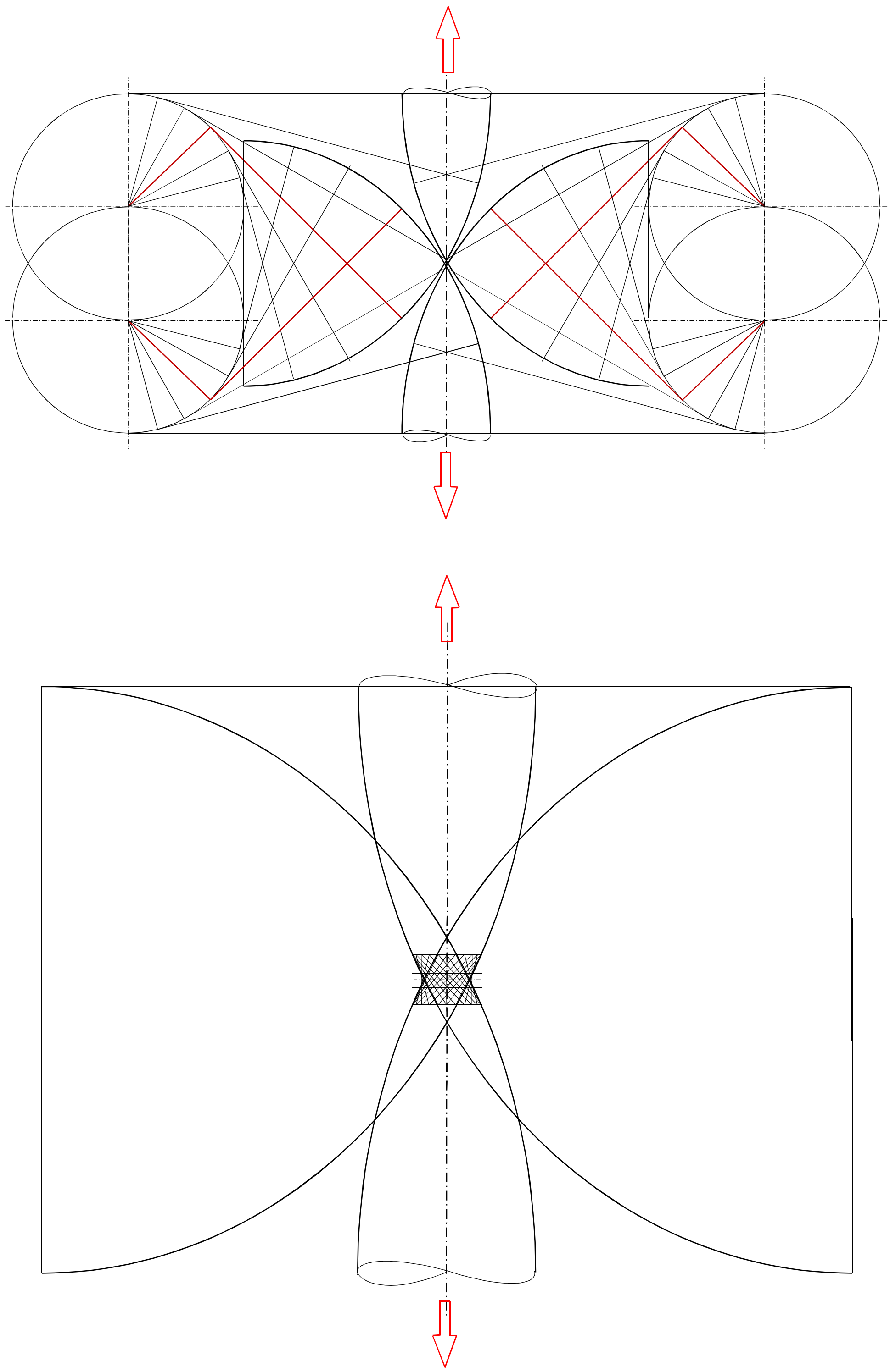


Fig.2

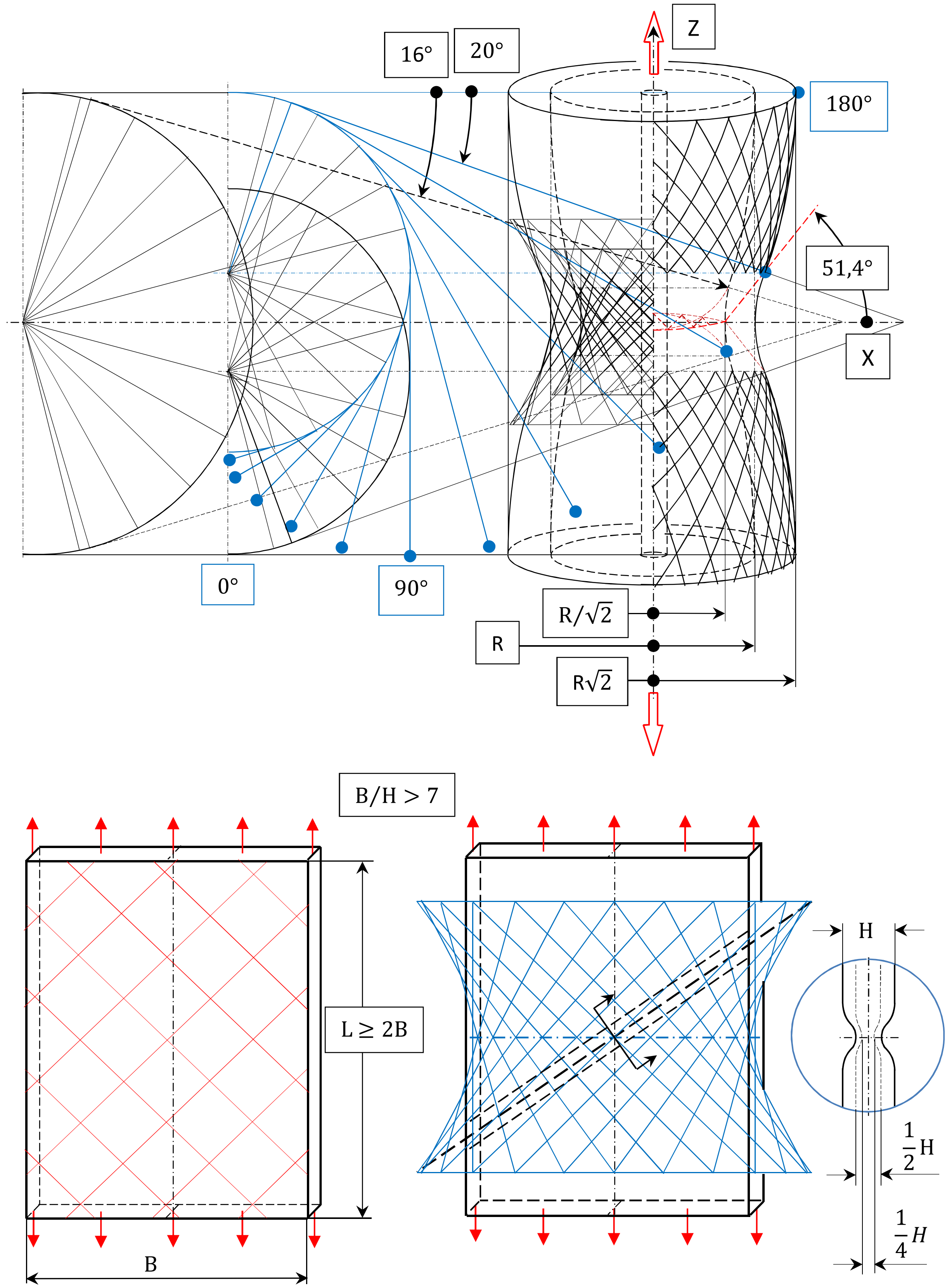


Fig.3

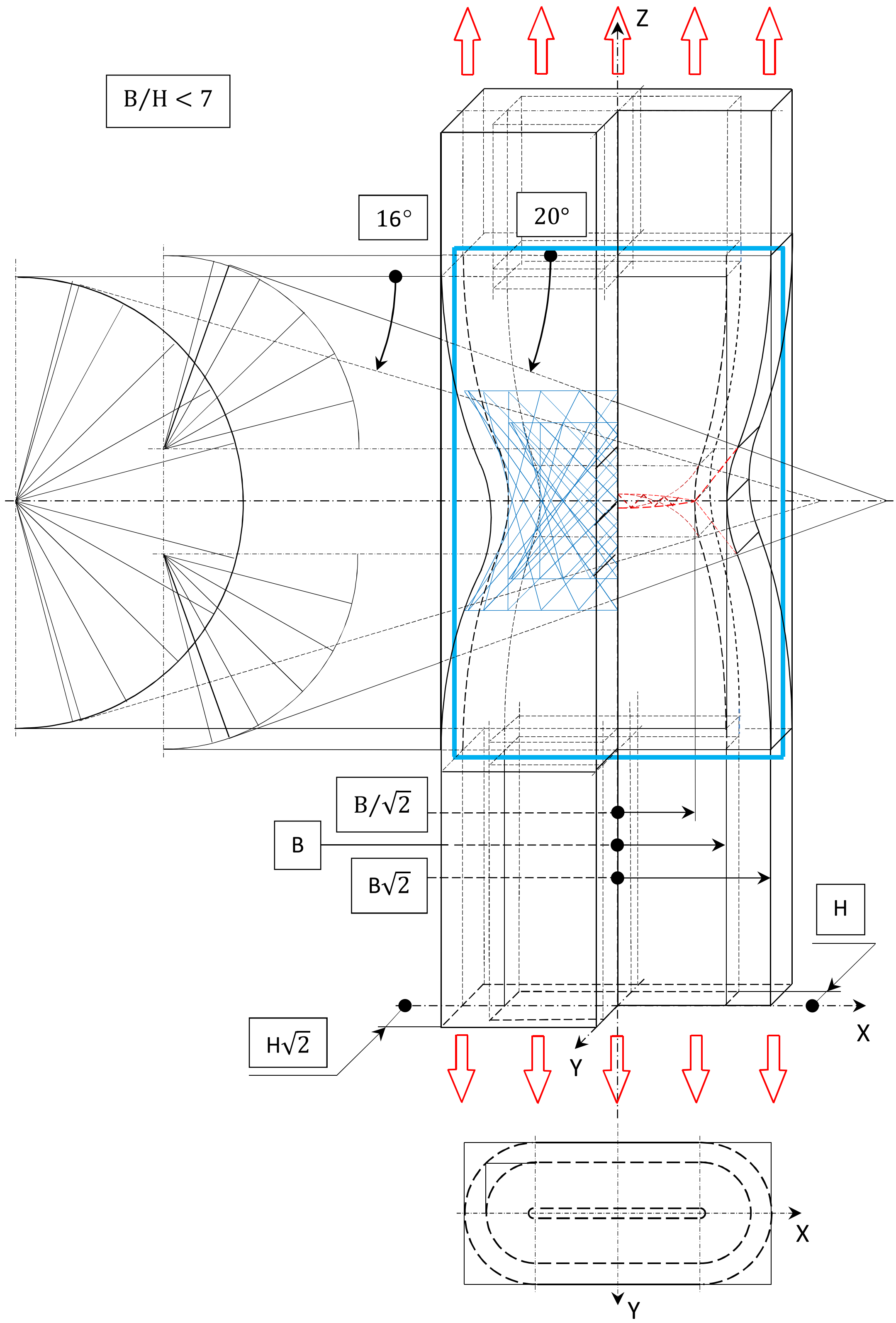
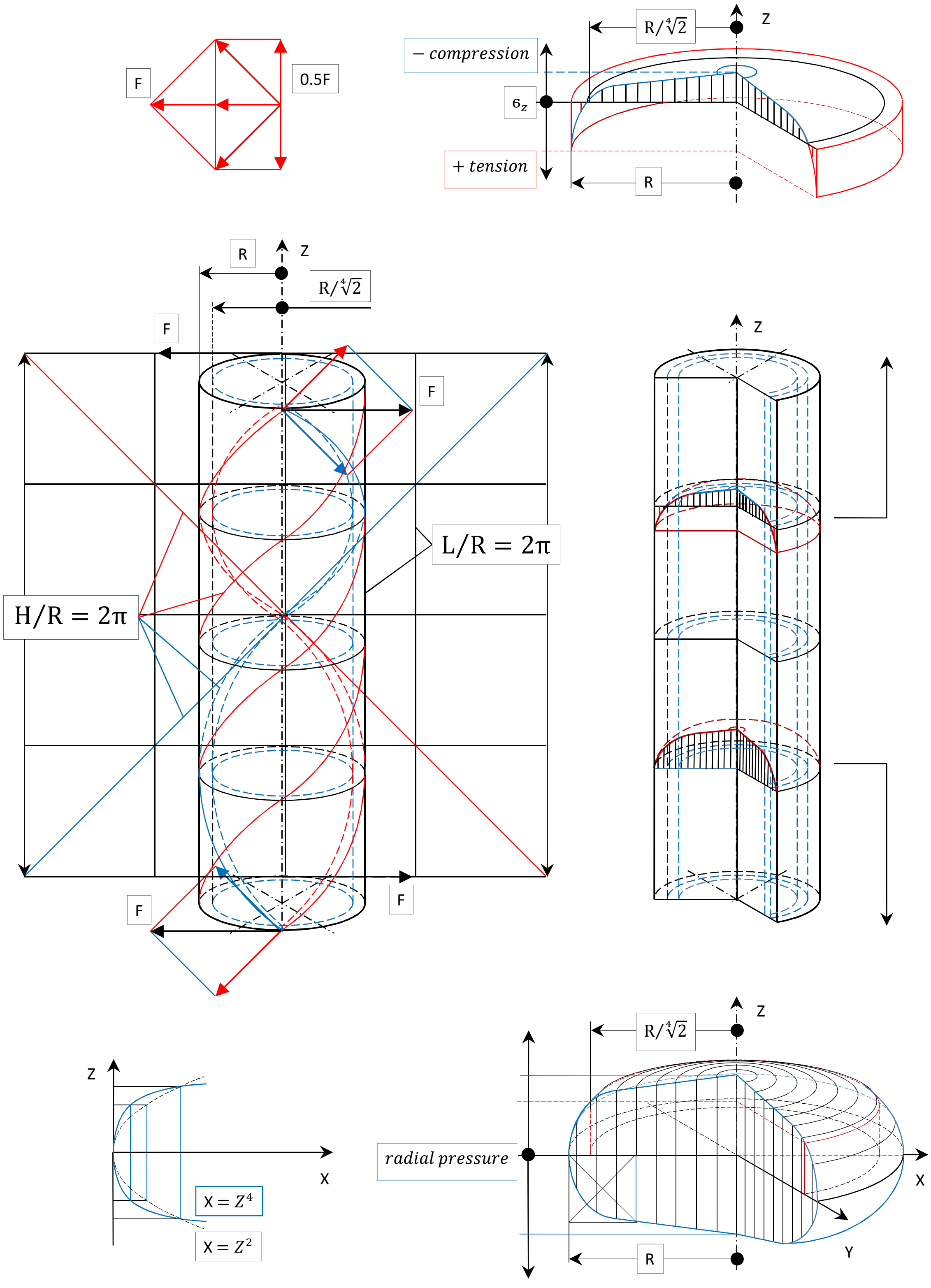


Fig.4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0.5F
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− compression
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R
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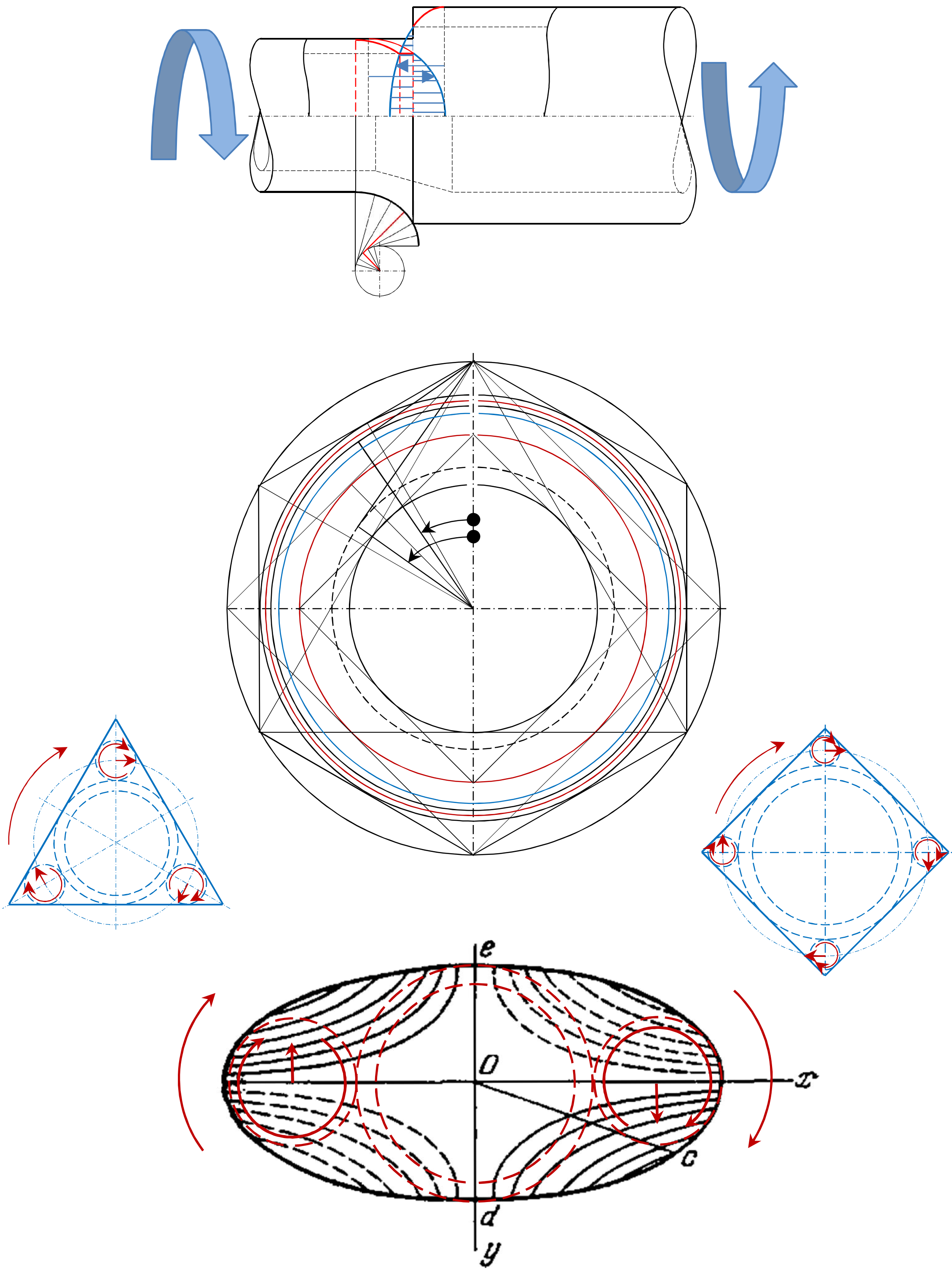
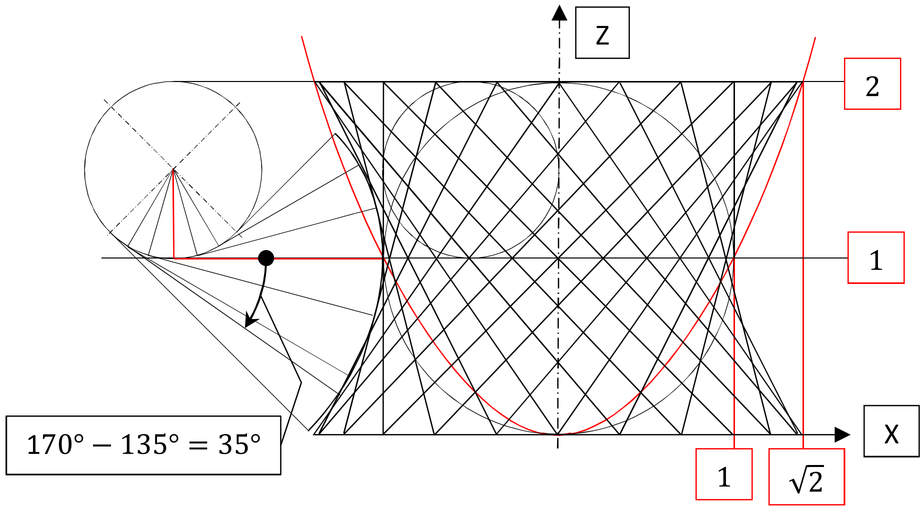
Fig.5



Black lines by St. Venant (1854) from [5, 6]

Fig.6. In the center: division of the circle area onto relative parts in consecutive order: 0.75; 0.707 (0841^2); polar moments of inertia of shell and core under torsion are equal); 0.67 (35°16′); 0.624 (contraction coefficient of free liquid jet outflowing of orifice); 0.5; 0.33 (54°44′); 0.25



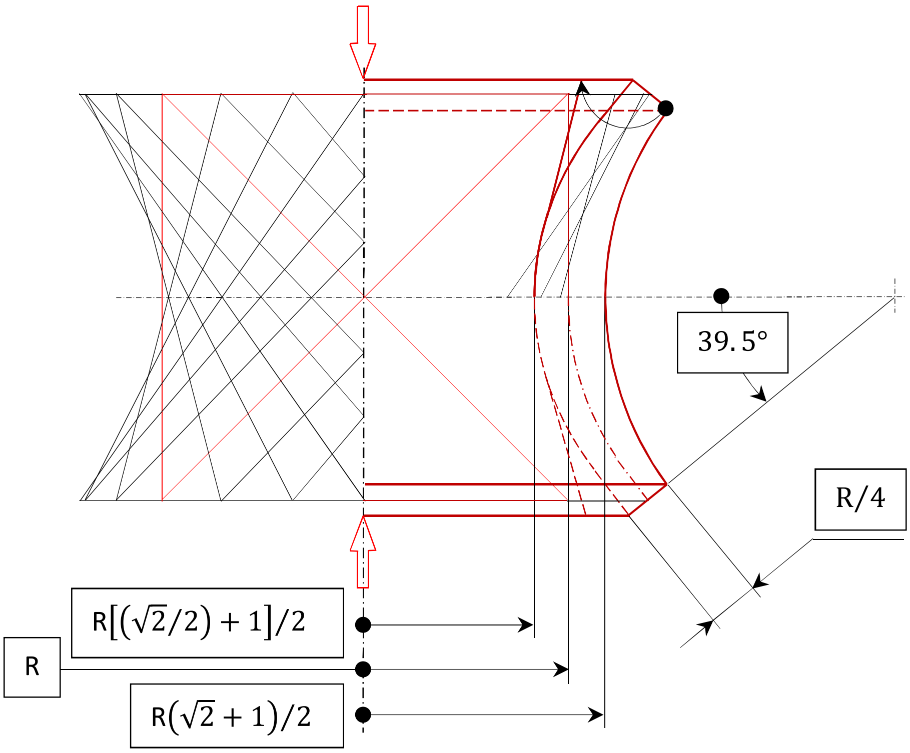


Fig.7

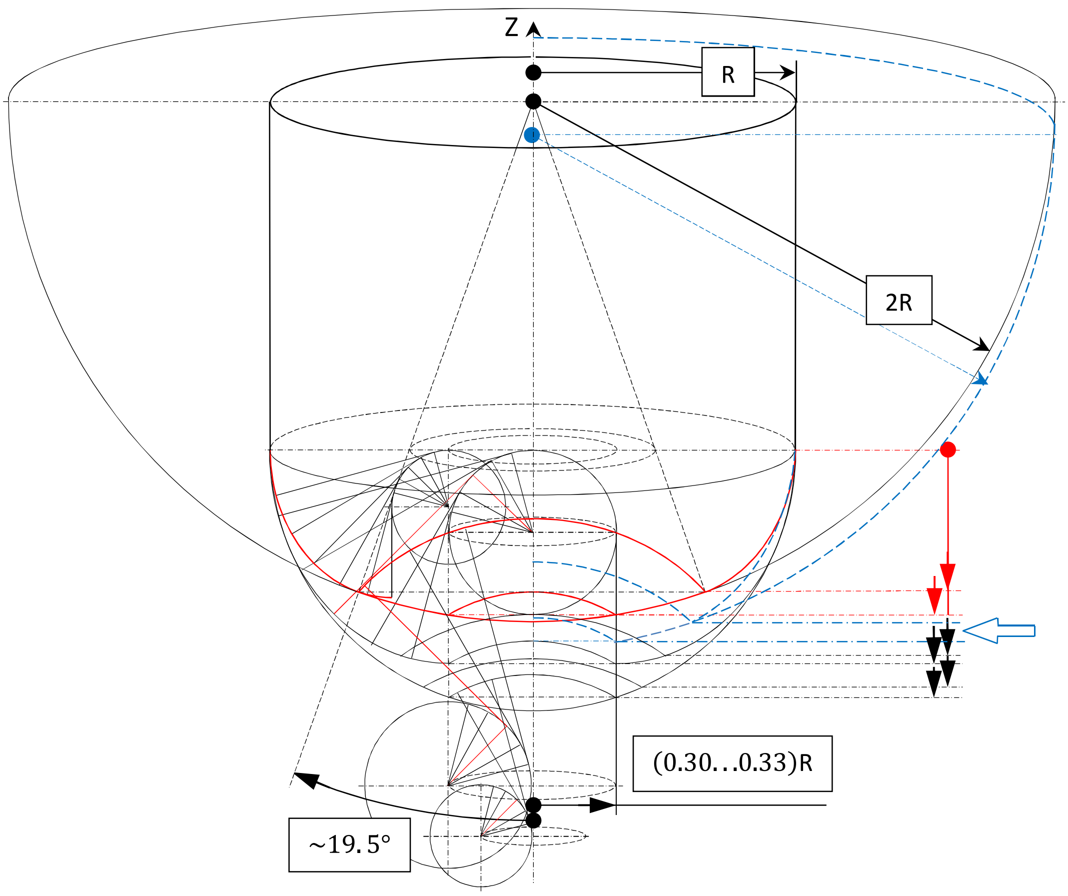
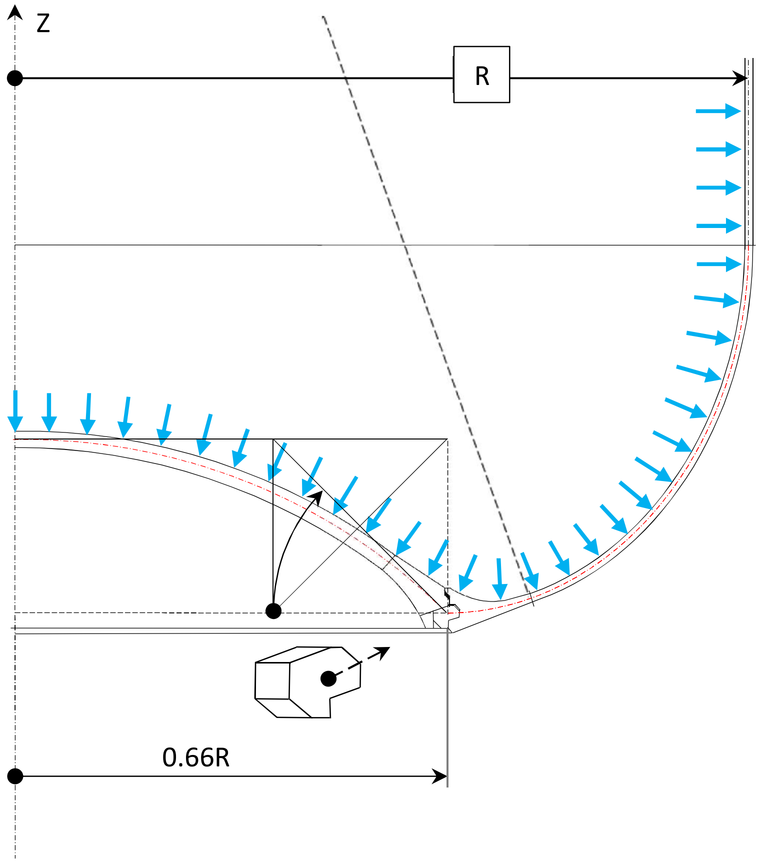


Fig.8



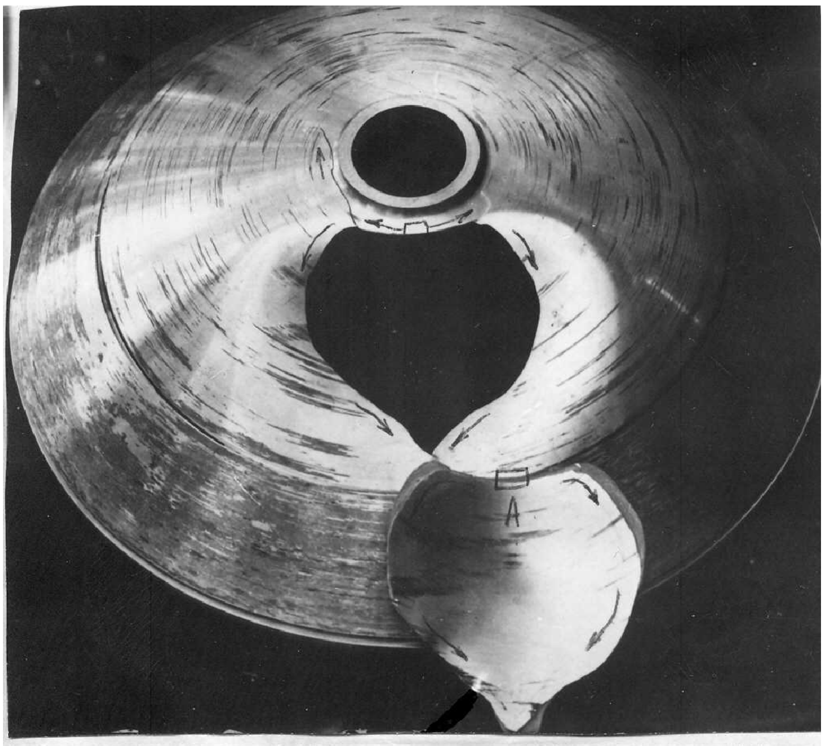


Fig.9

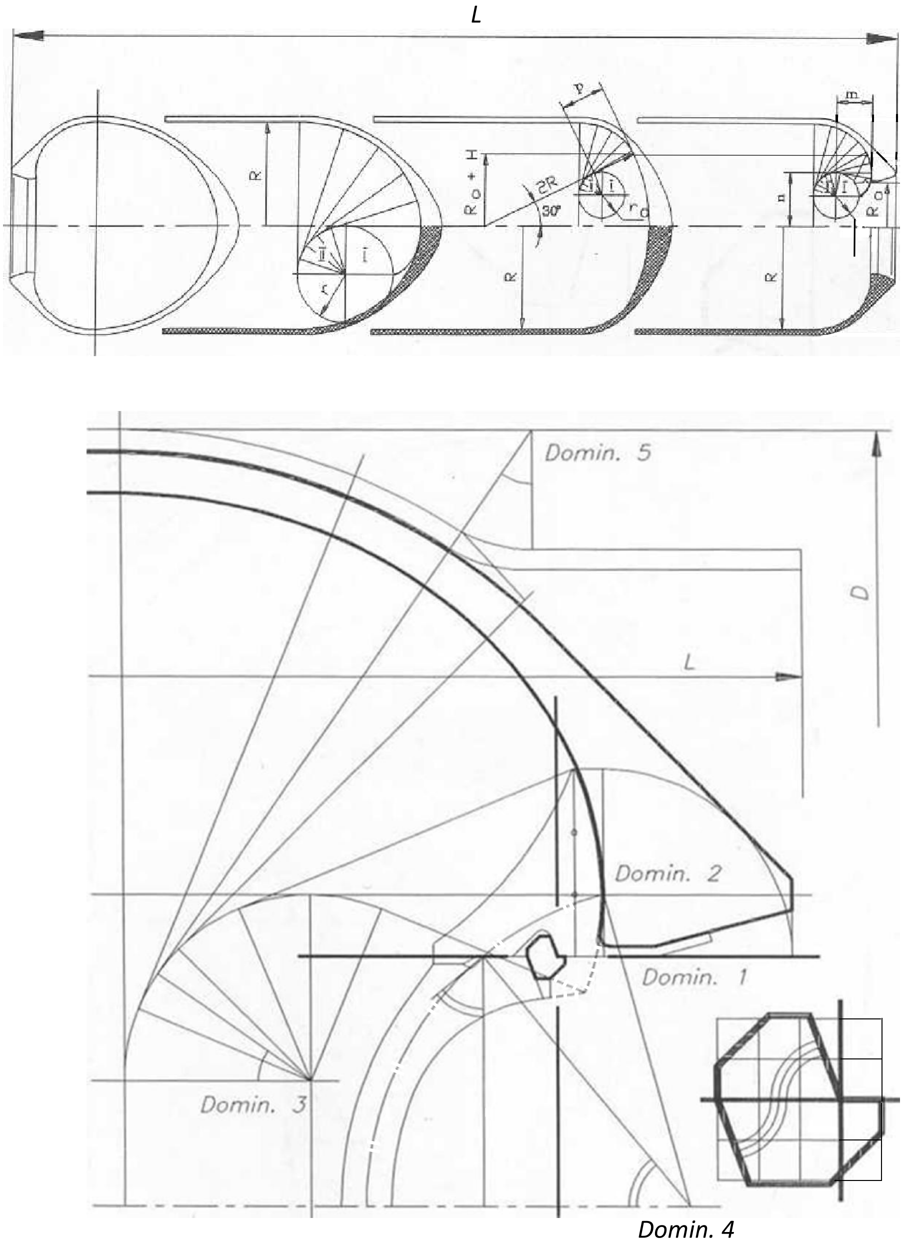
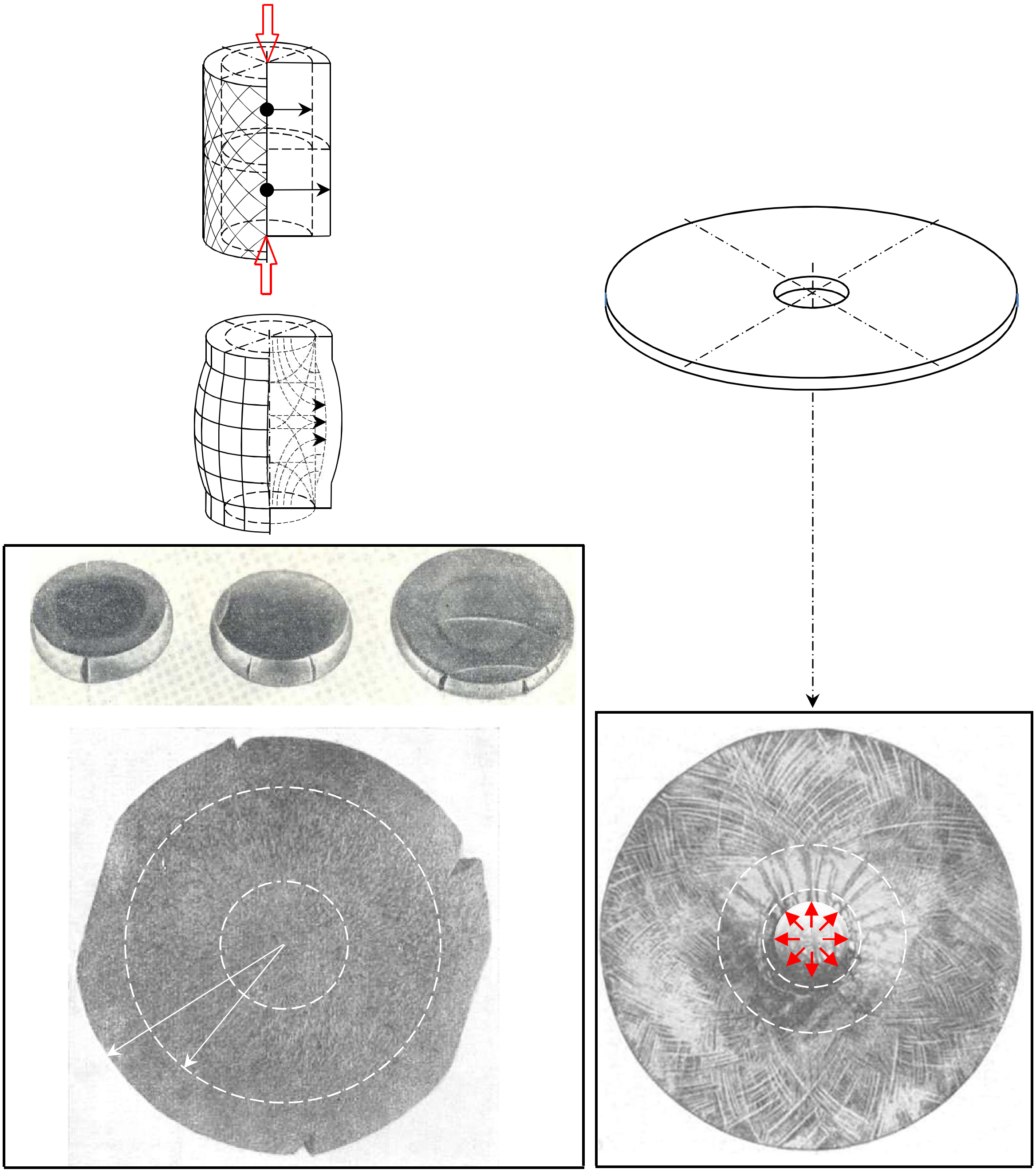


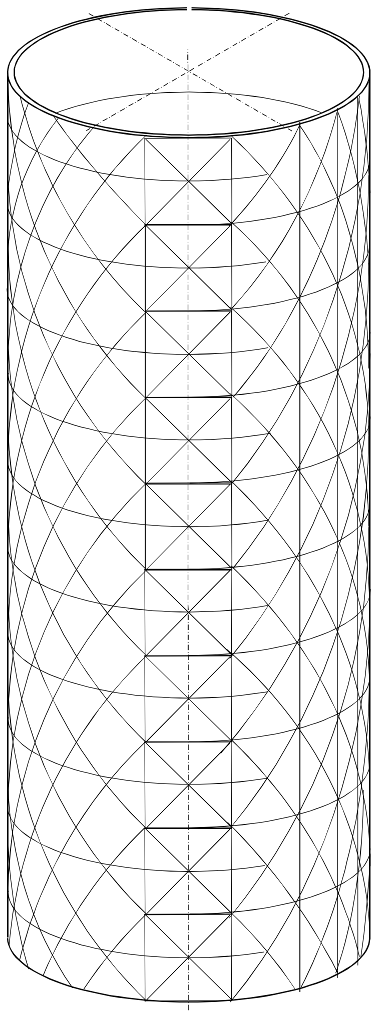
Fig.10



Photos by V.G. Osipov [9]

Photo by L. Hartmann (1896)
from A. Nadai [3]

Fig.11: in the left part, ratio of radii of shell and core parts in cylindrical specimen before testing remains after plastic deformation. Enlargement of the photo in the left lower part in ~2.5 times allows to see the tracks of radial plastic flow in the specimen core part. The right photo shows Luders tracks in kind of two sets of logarithmic spirals on surface of heavy-walled disk



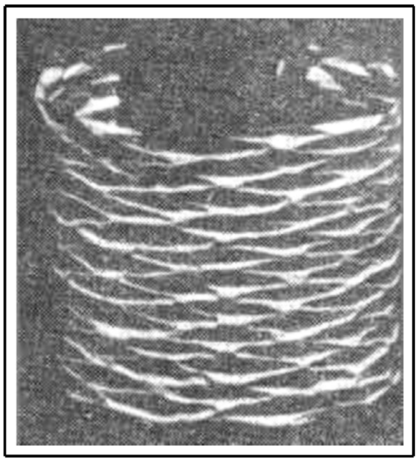


Photo by A.P. Coppa (SSL, General Electric, 1960) from [10]

Fig.12

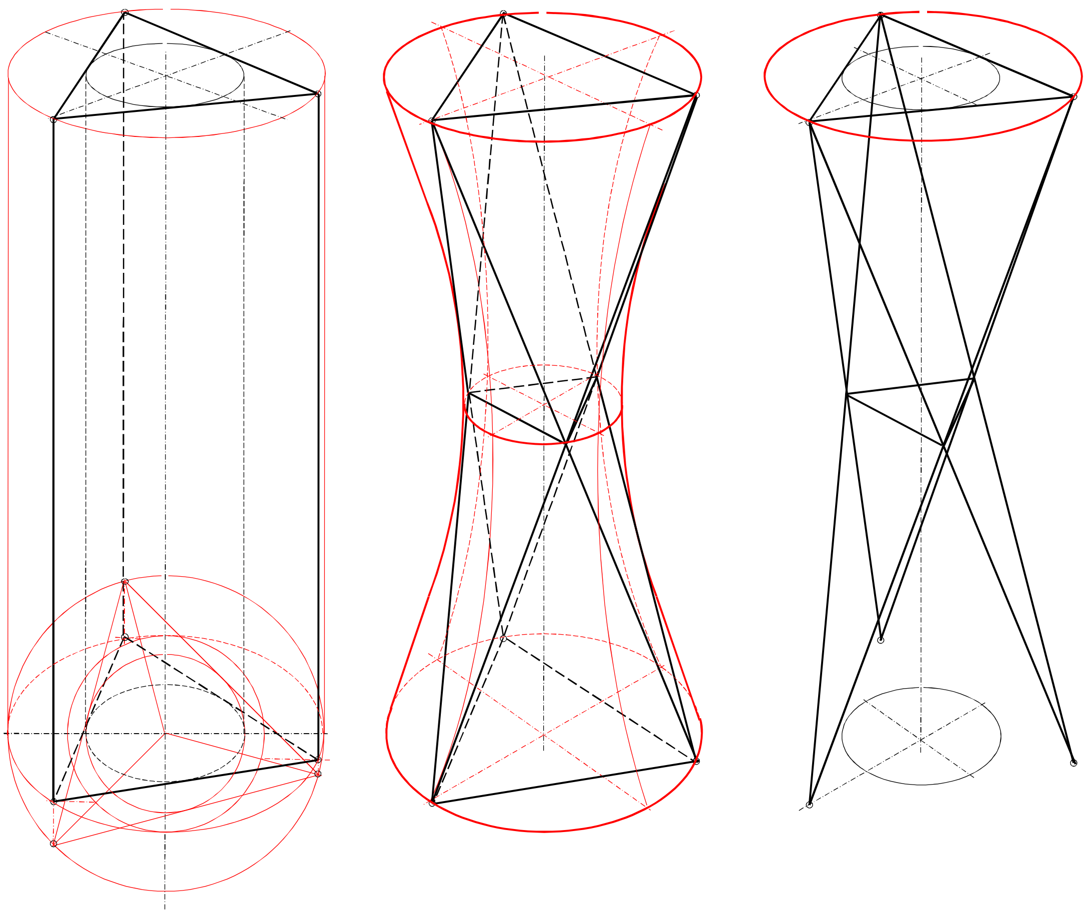


Fig.13

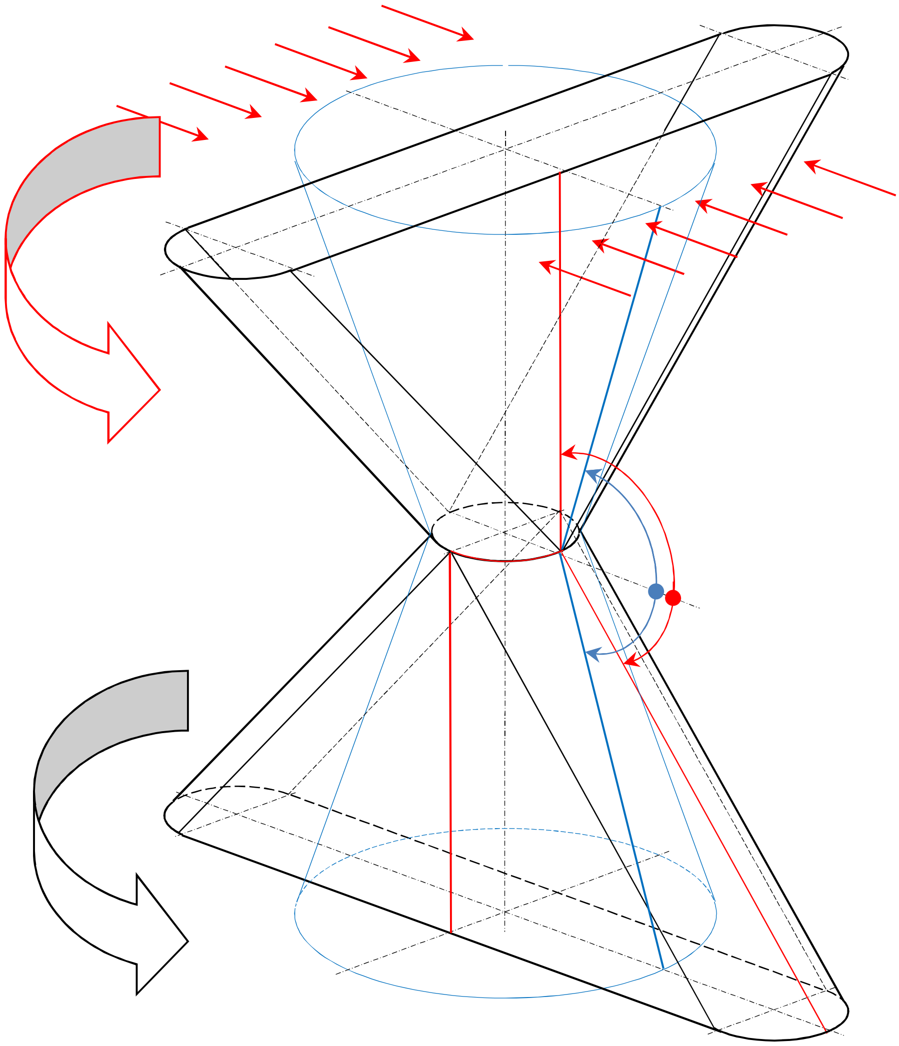


Fig.14