Mechanics of solid body, hydromechanics and gas dynamics in conic sections as a way for solution of the problems

Part 2: Kinematics of real fluid motion with transitional jets in pipe, orifice

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It is produced a solution of problems on kinematics of a real fluid motion in pipes with smooth and abrupt change in its cross-section including transitional converging and diverging jets and jet outflowing out of pipe, orifice. The solution in kind of synthesis of the flow field trajectories and the motive power field trajectories is obtained by means of graphic-analytic method in combination with plane curves and surfaces of rotation, formed by its, from set of conic sections, as instruments.

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Introduction

Subject of a given article is question of possibility and method for kinematic rational and visible reproduction of the flow and force fields for real fluid (water, air) in zones of smooth and abrupt change in area of a pipe cross-section and outflow through orifice in thin wall.
Solution in kind of synthesis of the flow field trajectories and the motive power field trajectories is obtained by means of graphic-analytic method in combination with plane curves and surfaces of rotation, formed by its, from set of conic sections, as instruments.
Textual part in the given article is offered in kind of commentaries to graphical solutions.

Solutions

Fig.1, in its upper part, shows two diagrams, conjugated by single axis: the left part is a form of cylindrical specimen from plastic metal with neck aroused in it under tensioning; the right part is one of stages of transforming a free falling thin compact water jet into serial falling drops. Such comparison shows that the surface tension forces in compact water jet under its tensioning by gravity force act in kind of two assemblages of helical lines – right- and left-handed – both inclined under 45° to the jet axis and mutually perpendicular each other and ones are trajectories of principal stresses. A uniformly accelerated motion of the jet leads to subsequent decrease of its diameter and correspondingly to an increase of principal stresses. Such smooth flow cannot be going on endlessly and one is broken by local narrowing jet in kind of neck The neck in the water falling jet takes a form of the right one-hollowed hyperboloid of rotation (just as in case of cylindrical metal specimen), where trajectories of principal stresses become straight lines. The surface neck is conjugated with the jet surface by means of evolventoid of rotation. The neck arises suddenly and decreases the jet cross-section area and correspondingly the jet flow rate in half. As a result the jet accumulates the first water droplet. Accumulation of the second droplet follows the first droplet fall etc. In that way the free falling smooth water jet transforms into series of the falling drops.

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In its middle the fig.1 shows Venturi tube, built by means of abovementioned hyperboloid and evolventoid of rotation, conjugated between itself and cylindrical parts of the tube.
The lower part of fig.1 shows graphical formula of geometrical compatibility applied to a circumference evolvent and a right one-hollowed hyperboloid profile in kind of equilateral hyperbola. Such compatibility of these curves and surfaces of rotation by ones determines possibility of graphical reproduction of a fluid movement trajectories applied to the axisymmetric flow systems i.e. to the problems of hydrodynamics at least.

Fig.2 offers graphical formula of geometrical compatibility for boundary line of a fluid flow in part of a flow system having an abrupt decrease in its cross-section area. A profile of the flow part is formed by contour ABCD which is axisymmetric relative to axis Z. Fluid moves from the left to the right. A copy of lower diagram out of fig.1 is placed here in point C, and one gives a part of boundary line of a flow contraction at inlet to narrowing part of the flow system. A circumference arch having its center P_{in} in axis Z and coming in to a point C in the limits 35° gives transversal boundary of contracting and accordingly accelerating part of transitional jet. Other transversal boundary of the jet part is minimum cross-section of the hyperboloid of rotation. In case of lack of contour CD we have outflow out of orifice in thin wall BC with its diameter equal to minimum cross-section of the hyperboloid. And in case of sufficiently large length of contour CD the described just now outflow will be supplemented with the expanded flow section in kind, for example, of the hyperboloid mirror reflection.
In upper part of the diagram it is adduced a mirror reflection of the circumference evolvent. A part of the evolvent, placed in red rectangle and increased with keeping its proportions in scale $(R_1 - R_2)/h$ is a boundary line of transitional jet, which is formed in front of the abrupt decreased cross-section area in the flow system. Contour AB is a tangent to this boundary line and the line comes to a point C as normal to circumference arch 35°; at the same time, of course, the arch radius $F_{in}C$ is tangent to the boundary line. This arch 35° shows that the jet cross-section has spherical but not plane form, and one makes together with the left side of the half-hyperboloid a plano-convex hydraulic lens. In the given case the lens is a converging (normal) inlet lens. And a point F_{in} is a focus of the hydraulic lens. The jet contraction coefficient in such physically correct form exceeds its traditional quantity 0.624 by 10 per cent, and it equal 0.686. The diagram, side by side with this, determines ratio of radii $(R_1/R_2) \leq 2.9$ as a flowing system, element, otherwise we have dealings with the tank-pipe system. In this way we can distinguish transitional jet motion zone from zones of a fluid stagnation or local circular near-walled motion.

Fig.3, in its upper part, shows next graphical formula of geometrical compatibility of a circumference evolvent and a right one-hollowed hyperboloid of rotation.
The question is: what is point in verifying compatibility of the hyperboloid profile with evolvent? The author is periodically inserts the hyperboloid fragment side by side with the transitional jet boundary line in kind of evolvent, since the hyperboloid structure reminds to courteous reader that motive forces in a fluid stream, including transitional jet, act along trajectories in kind of two mutually perpendicular assemblages of coaxial helical vector lines – dextrorse end sinistrorse - under 45° to its axis; and geometrical sum of the motive forces determines not only a forward fluid flow but also all other kinds of interaction of a fluid stream with the flow system. In this way the force field governs the field of the fluid motion trajectories.
A picture in middle part shows a diagram of building the flow field trajectories in a flow element with abrupt decrease in its cross-section at $(R_1/R_2) = 2.9$. We divide cross-section of both parts

of the flow element into annular layers, for instant 10, with equal quantity of its cross-section area by simple succession:
$R_{11} = 1$; $R_{12} = \sqrt{0.9}$; $R_{13} = \sqrt{0.8}$; ... $R_{110} = \sqrt{0.1}$; and accordingly
$R_{21} = 1$; $R_{22} = \sqrt{0.9}$; $R_{23} = \sqrt{0.8}$; ... $R_{210} = \sqrt{0.1}$.
Thus the transitional jet structure is presented by a sum of annular elementary layers (but not by a sum of elementary jets) placed concentric to each other and to an axisymmetric flow element, system.
Correspondingly to the flow element geometry the transitional jet motion has also two sections, restricted in the diagram by three vertical red lines: the first is section of acceleration and the second is section of deceleration.
In the first of these sections the boundary trajectory of transitional jet in kind of arch of evolvent walks smoothly away from wall in the flow element section R_1 and, passing through edge, R_2, reaches its maximum contraction in inlet part of the flow element section R_2. The boundary trajectory of transitional jet consists of two arches of the same evolvent – large and its decreased copy – and the second of their is reflected from left to right and from top to down relatively to the first. These two arches form a pair describing smoothly the boundary trajectory of transitional jet in section of its accelerated motion.
Conservation of proportions at every change in scale of the evolvent arches and the hyperboloid profile is indispensable condition for carrying-out correct graphical constructions.
A constructing the transitional jet boundary trajectory in the second section, as section of the jet deceleration and its expansion, is quite similar to above described for the first section; features in this case are in using the second plano-convex hydraulic lens (an exit lens) with its focus in point F_{ex} and also in that the section length depends on the jet velocity: at not very high velocity the length is minimum and one has its profile in kind of the hyperboloid profile and at high velocity the length is increased and then one takes a kind of the evolvent arch. The jet boundary trajectory continues smoothly the boundary trajectory of the first section and reaches wall R_2 in the limits 35°.
Construction of intermediate trajectories of transitional jet is quite similar to above described for the boundary trajectories in both sections of the transitional jet motion.
The lower diagram in fig.3 shows a similar flow element at $(R_1/R_2) < 2.9$. In this case an incline of the jet boundary trajectory at inlet to the second part R_2 of the flow element is 25°. Since the angle is less than 35°, the flow is called a straitened flow, and such flows arouse the near-walled vortical motion. The diagram shows that contraction in transitional jet is decreased as ratio R_1/R_2 is decreased. At the same time the well-known numerous experiments showed that the contraction coefficient as a ratio of areas of the jet and tube (orifice) cross-section is decreased as Reynolds number is increased.

Fig.4, in its middle part, shows a diagram in black lines, composed by A.D. Altshul (1962) [1], where μ is a flow rate coefficient, φ is a velocity coefficient and ε is a contraction coefficient as functions of Reynolds number. Author of the given article has plotted on the diagram two parabolas in red and blue points corresponding to the diagram experimental points. Combination of these parabolas shows distinctly a sharp change in the transitional flow character at Re = 350. Upper part of the fig.4 shows a decrease of the contraction coefficient as Reynolds number is increased by means of the hyperboloid scale – with keeping it proportions.

Lower part of the fig.4 shows variants of the transitional jet contraction and subsequent flow in tube, short mouthpiece and abrupt local narrowing spot.

Fig.5 shows a profile ABCDEFG of axisymmetric flow system. Blue evolvent AC is a boundary trajectory of transitional jet and black evolvent A'C' equidistant to it is a pressure profile of the jet onto motionless fluid surrounding it. Conservation of constant quantity of the pressure - with its quantity AA' - along section AB (R_1) is stipulated by combination of three factors:
- a decrease of a curvature radius of the jet boundary trajectory - evolvent AC - as it comes nearer to inlet edge of section CD (R_2) ;
- smooth decrease of static pressure up to zero in point C along parabola in second power with its axis coincided with axis Z; the parabola (with its part A"C) is showed by black dashed line;
- smooth increase of a velocity head component directed along normal to the boundary trajectory of transitional jet which is showed by red dashed straight line.

In a result it is going on the following:
- the largest quantity of the evolvent curvature radius stipulates negligibility of change in initial parameters of transitional jet from its beginning up to middle of section R_1;
- noticeable and progressive change of these parameters is going on at second half of section R_1, when static pressure begins to follow the second power parabola and the velocity head component, directed along normal to the jet boundary trajectory, begins to follow straight line;
- there exists a place where static pressure and the velocity head component both equal to each other and in sum ensure constant quantity of the jet pressure onto surrounding it motionless fluid;
- in point C static pressure becomes equal to zero, and the velocity head component, increasing in its quantity, ensures by itself constant pressure of the jet onto motionless fluid in this point.

Explanation of such, unusual at first sight, situation is stipulated by the following. The velocity head vector is always directed along the transitional jet motion axis, and components of the vector are directed one - along a normal and the other - along a tangent to the jet boundary trajectory, which one is arch of evolvent with quite intensive decreasing in its curvature radius as far as this curve comes in to a point C. While the normal component exerts pressure onto the surrounding motionless fluid, other component of the velocity head vector, tangent to the jet boundary trajectory, is directed to the flow axis Z, i.e. away from the motionless fluid. Such situation is remained when $(R_1/R_2) \geq 2.9$ and the evolvent angle is 35°. Otherwise transitional jet arouses secondary vortical motion in a near-walled fluid.

Action of static pressure is full stopped in point C in which the parabolic curve intersects the jet boundary trajectory.

The adduced geometrical constructions and textual explanation of constancy in the pressure quantity of transitional jet onto surrounding it motionless fluid are ocular evidence to theorem: evolvent, which is equidistant to the given evolvent, can be reproduced by combination of straight line and arch of parabola in second power.

In inlet part of section R_2 (point C) the boundary trajectory of transitional jet follows the hyperboloid profile and forms a converging-diverging separation zone of the jet boundary trajectory away from wall. A change of static pressure in the converging part of transitional jet is determined mainly by change of the velocity head - from quantity of its tangent component applied to spherical surface of inlet hydraulic lens to its quantity in a narrowing spot cross-section - inversely proportional to the jet contraction coefficient (equal 0.686). Minimum quantity of negative static pressure obtained by such way and the narrowing spot cross-section as neck of the hyperboloid allow to insert parabola in second power with its axis coincided with the cross-section plane, with its apex coincided with the minimum pressure and also with smooth contact of its left branch with previous parabola in point C. In that way the just now inserted parabola reproduces the negative pressure curve all over the separation zone at not very high velocities of transitional jet.

Positive static pressure in the section CD, behind the separation zone (neck) of transitional jet, is determined usually by Darcy-Weisbach formula.

Subsequent flow in sections $R_2$and R_3 is showed in diagram placed below. This part of the flow system has abrupt expansion $(R_2/R_3) \leq 2.9$. Corresponding to it the boundary trajectory of transitional jet has a kind of the same evolvent arch lesser than 35° which one is decreased in its scale. A constructing the pressure curve in this case it is necessary to carry-out with taking into consideration of a decrease of the velocity head vector proportional to $(R_2/R_3)^2$. We suppose that the pressure curve must be the evolvent equidistant to the boundary trajectory of transitional jet in this section, and therefore one ensures constant pressure of transitional jet onto surrounding it near-walled fluid, as it was above in section R_1. In order to make sure that it is so, we connect an end of the velocity head component directed along normal to the jet boundary trajectory with evolventic curve of the trajectory by straight line tangent to latter (showed by red dashed line). After that it should be placed a parabola in second power with its axis perpendicular to axis Z and coincided with inlet cross-section of section R_3 and with its apex coincided with the line of static pressure in outlet of section R_2. There is only to set a scale of the parabola so as to balance curvilinear clearances on both sides of pair of the evolvents, formed by above straight line and the parabola.

A diverging axisymmetric nozzle profiled by evolvent ensures the most uniform deceleration of transitional jet with sufficiently low hydraulic losses.

Specificity of interaction of the expanding (decelerating) transitional jet with surrounding it fluid is in that such jet arouses in the latter secondary vortical motion, and this phenomenon is going on not only in pipes at sufficiently sharp increase or decrease in its cross-section area but also at outflowing the jet into unrestricted homogeneous with it fluid medium. In contrast to it, the contracting (accelerating) transitional jet at inlet part of a flow element with quite abrupt decrease in its cross-section area not arouses secondary motion in surrounding it near-walled fluid.

Cause of such difference is in that in the first of these two cases the velocity head component directed along tangent to the boundary trajectory of transitional jet is directed to the flow element wall and thereby one stipulates the flow bifurcation with recurrent secondary near-walled flow; intensity of the flow is proportional to an incidence angle in the limits 35°. And in the second case the velocity head component directed along tangent to the boundary trajectory of transitional jet is directed away from the wall and thereby one arouses separation of the jet flow from the flow element wall not exciting secondary flow in the separation zone.

Thus transitional jet, which increases its axial velocity head when one is accelerated and which decreases its axial velocity head when one is decelerated, the jet conserves constant quantity of its initial pressure onto surrounding it motionless fluid medium.

Another specificity of transitional jet - in section of its separation away from a wall - is that at decreasing of static pressure in it up to lower atmospheric pressure during its spontaneous contraction the jet remains always its continuity, while in Venturi tube a stream motion, forced to follow converging-diverging section of the tube, can be accompanied by cavitation.

Fig.6 offers two examples of the rational flow element $(R_1/R_2) = 2.9$ with using evolventic trajectories.

Fig.7 shows free outflowing out of orifice in thin wall at $(R_1/R_2) = 2.9$ and nozzle profiled by evolvents.

Fig.8, in its upper part, shows the flow element with abrupt increase of its cross-section radius ratio $(R_1/R_2) = 0.5$. Transitional jet is constructed by means of evolventic curves, and one is still axisymmetric. The jet forms secondary circulating motion in surrounding it fluid. The motion is mainly beginning in zone of near-walled bifurcation of transitional jet. The jet trajectories are constructed by the evolvent combinations.
Lower part of Fig.8 shows outflowing of liquid (water) out of centrifugal sprayer. This diagram in black lines is adopted from M.S. Volynsky book [2]. Blue lines in the diagram give correct image of the jet, which one consists two parts: in its beginning the jet has kind of compact thin-walled right one-hollowed hyperboloid of rotation and subsequent motion in kind of conic jet consisting of a fine-dropped water aerosol.

Fig.9, in its upper part, shows a profile of the visualized field of velocities in water outflowing out of a head-water to a tail-water. This experimental result is obtained by M.S. Fomitchev [3]. A lower diagram offers two variants of a flow field structure in above head-water: with replenishment of it along horizontal from the left and from two transversal sides. The flow field structure contains a central trajectory of the flow and boundary trajectories determining zones of the flow stagnation near bottom and near wall in the head-water. A flow field structure in tail-water is quite similar that showed in upper part of Fig.8, and for constructing it author of the given article has used evolventic arches only changing its scale with conserving in proportions and changing its length only.

Fig.10, in its left part, offers a fragment of a cinema film compiled by set of the selected consecutive shots on confluence of two equal portions of epoxy resin suspended in water, adduced Ya.Ye. Geguzin book [4]. Before its contact these portions had right spherical form. The first contact means origin of single axisymmetric body, i.e. body of rotation, which has two polar spheroidal segments in its ends and lateral bispheroidal surface in contrast to solitary spheres every of which has limitless number of its poles and axis of symmetry, i.e. rotation with indeterminate structure of trajectories of surface tension forces. A diagram in the right part of fig.10 offers some consecutive stages of the confluence, constructed by author of the given article. It should be supposed that every sphere and spheroid contains in itself a core and shell parts; concentric circumference in dashed line is a boundary between these parts.
At once after contact, these two spheres quickly confluent on length of one thickness of its shell with forming between itself a crosspiece in kind of hyperboloid; now the single body of rotation consists two the same parts, and every of it has kind of a spheroid belt with adjoined to it spheroidal polar segment, which ones are connected by the crosspiece; and now structure of trajectories of surface tension forces becomes quite determined: in every of the polar segments these trajectories are its meridians; in every of spheroidal belts these trajectories are two sets of geodesic lines formed by rotation clockwise and anticlockwise of two cross-section planes, inclined to axis Z under $\pm 30°$ correspondingly; in the crosspiece these trajectories are formed by rotation clockwise and anticlockwise of two set of straight lines, inclined to axis Z under $\pm 60°$.

At consecutive stage, these spheroidal parts interflow on length of two thicknesses of its shell, and cores of these parts come in to contact. Now the spheroidal segments are increased and spheroidal belts become symmetrical; trajectories of the surface tension forces in polar segments as before are its meridians; two sets of geodesic lines on both spheroidal belts are inclined to axis Z under $\pm$ 45°.
A beginning of confluence of the core parts is accompanied by transforming the crosspiece surface into the right one-hollowed hyperboloid of rotation, connecting smoothly both spheroidal parts of the body.
When confluence of the core parts is reached on length equal thickness of the shell part, the body takes shape of cylindroid with spheroidal ends. Trajectories of the surface tension forces on polar segments are remained in kind of meridians; the trajectories on lateral surface of the body are also remained in kind of two sets of geodesic lines under $\pm$45° to axis Z.
It should be noted that the diagram is simplified since author of the given article not kept strictly condition for the body volume to be constant on some intermediate stages - in reality consecutive decrease in the body length is accompanied by increase in it cross-section.

Fig.11, in its right part, offers a diagram, combined with the above adduced known cinema film shots. The diagram shows that one of last stages before formation of single summary sphere can be offered in kind of evolventoid of rotation constructed by combination of four copies of the curve arch in the limits of its second quarter.

Final remarks
Author dedicates the given article to memory (445 years) of Johann Kepler, who had laid down the foundations of kinematics and for the first time had used the conic sections in mechanics.

Acknowledgements
Author expresses his deep gratitude to his son Alexey for help and supporting.

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[1] A.D. Altshul, Hydraulics and aerodynamics, Stroyizdat Publishing, Moscow, 1975
[2] M.S. Volynsky, Extraordinary life of ordinary droplet, Znanie Publishing, Moscow, 1986
[3] M.S. Fomichev, Experimental hydrodynamics of YaEP, EnergoAtomIzdat Publishing, Moscow, 1989
[4] Ja.Je. Geguzin, Droplet, 2^{nd} Ed., Nauka Publishing, Moscow, 1977

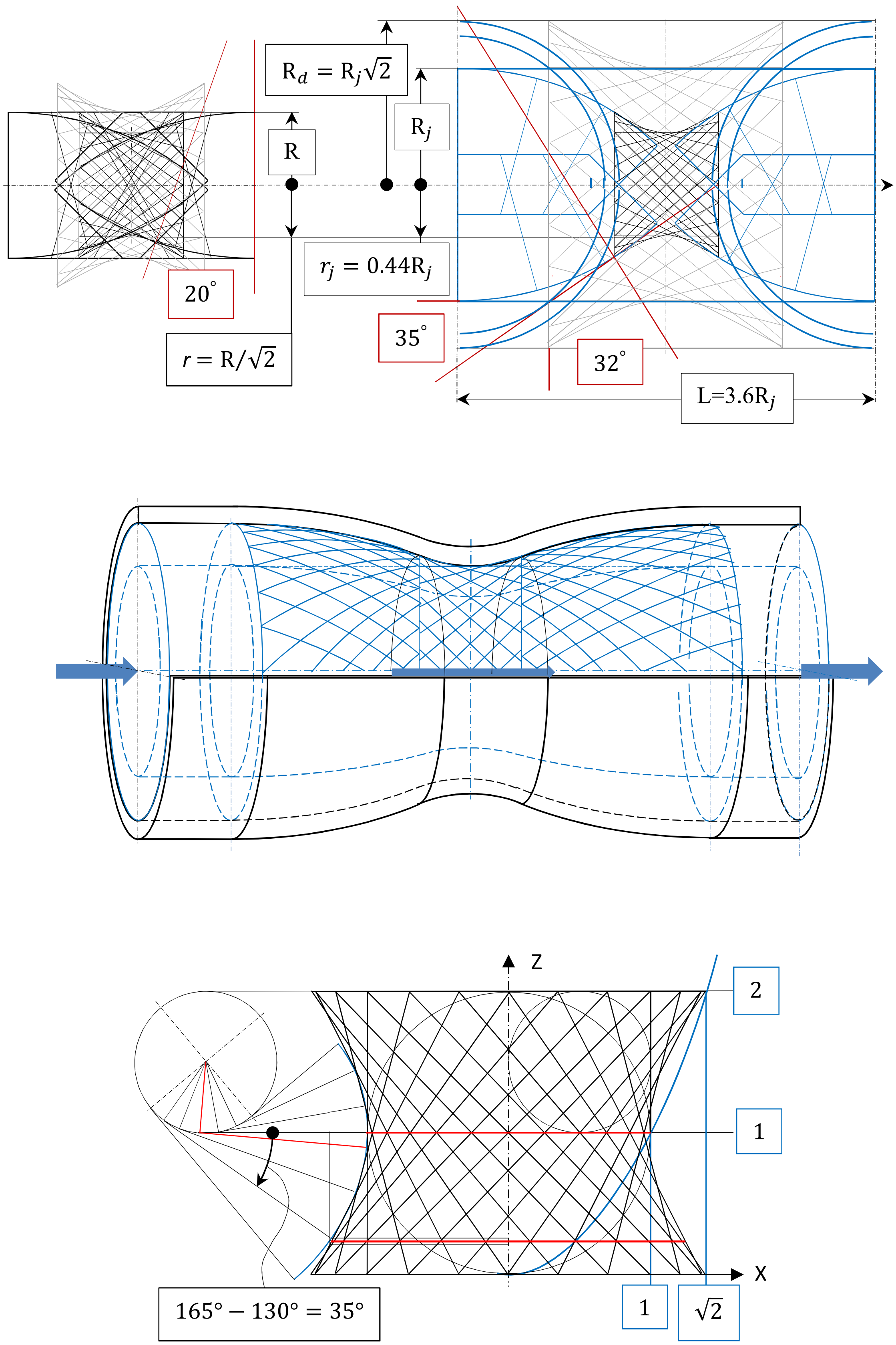

$R_d = R_j\sqrt{2}$
R_j
R
20°
$r = R/\sqrt{2}$
$r_j = 0.44R_j$
35°
32°
$L=3.6R_j$
Z
2
1
X
1
$\sqrt{2}$
$165^\circ - 130^\circ = 35^\circ$

Fig.1

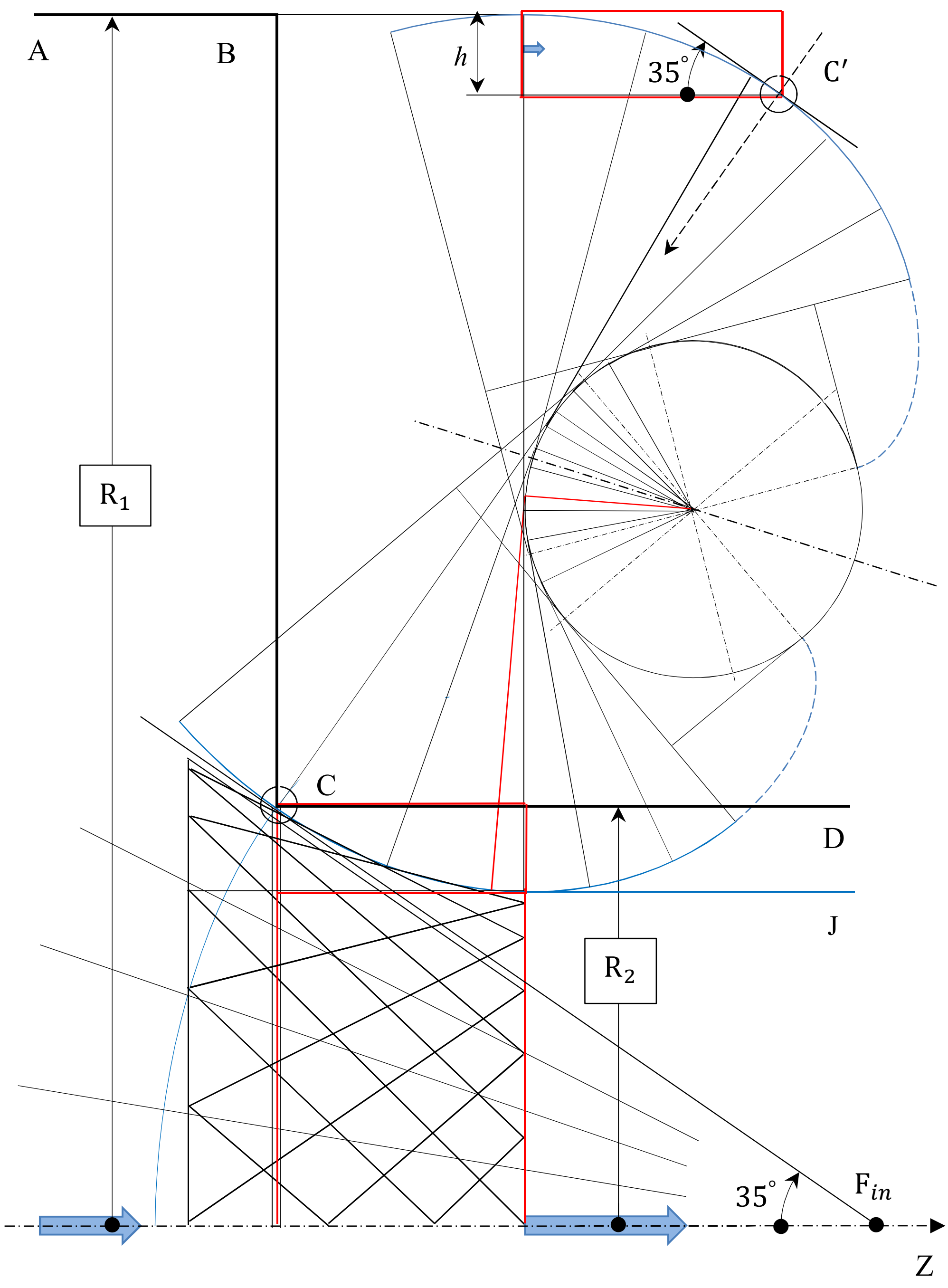


Fig.2

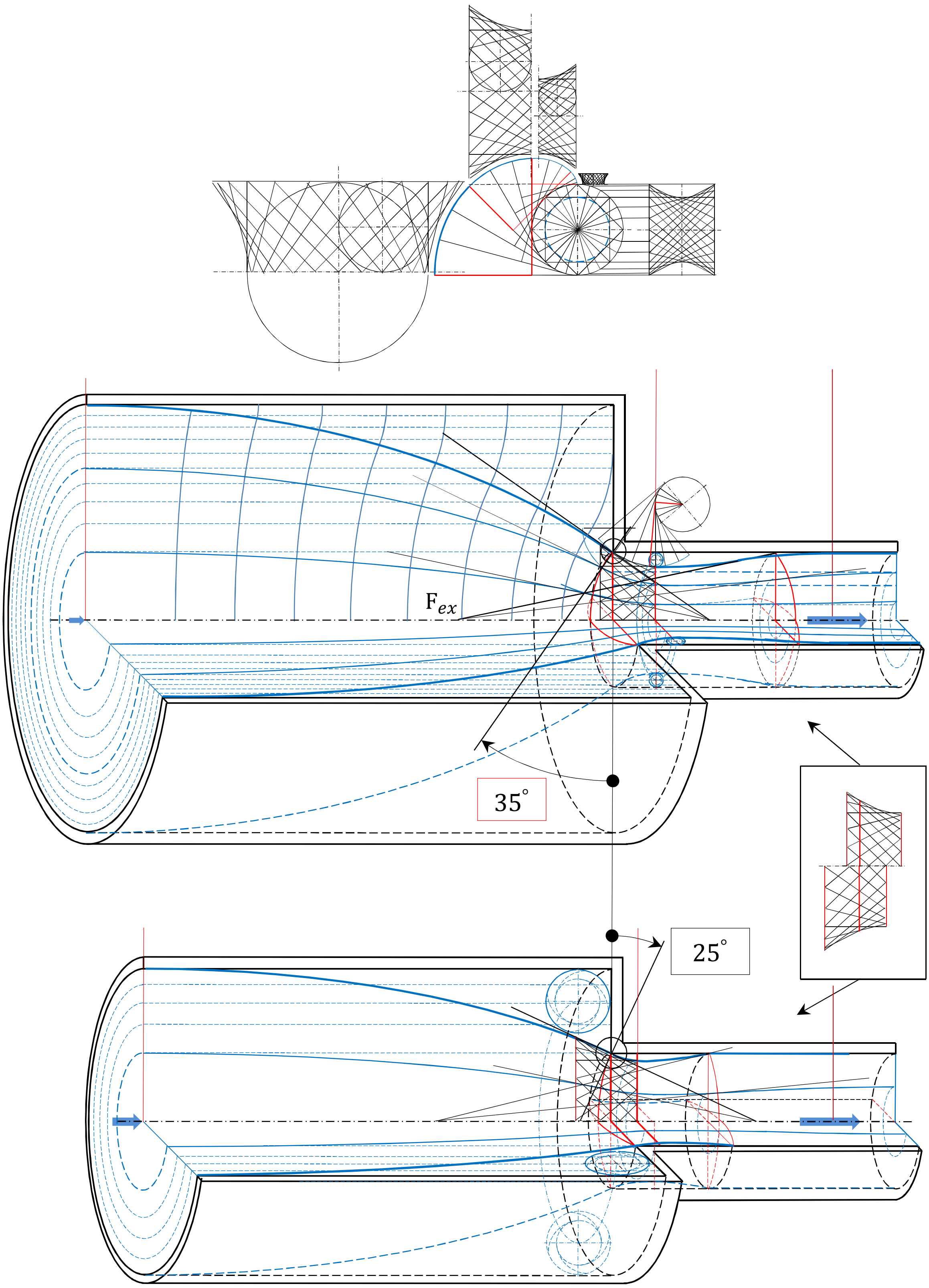
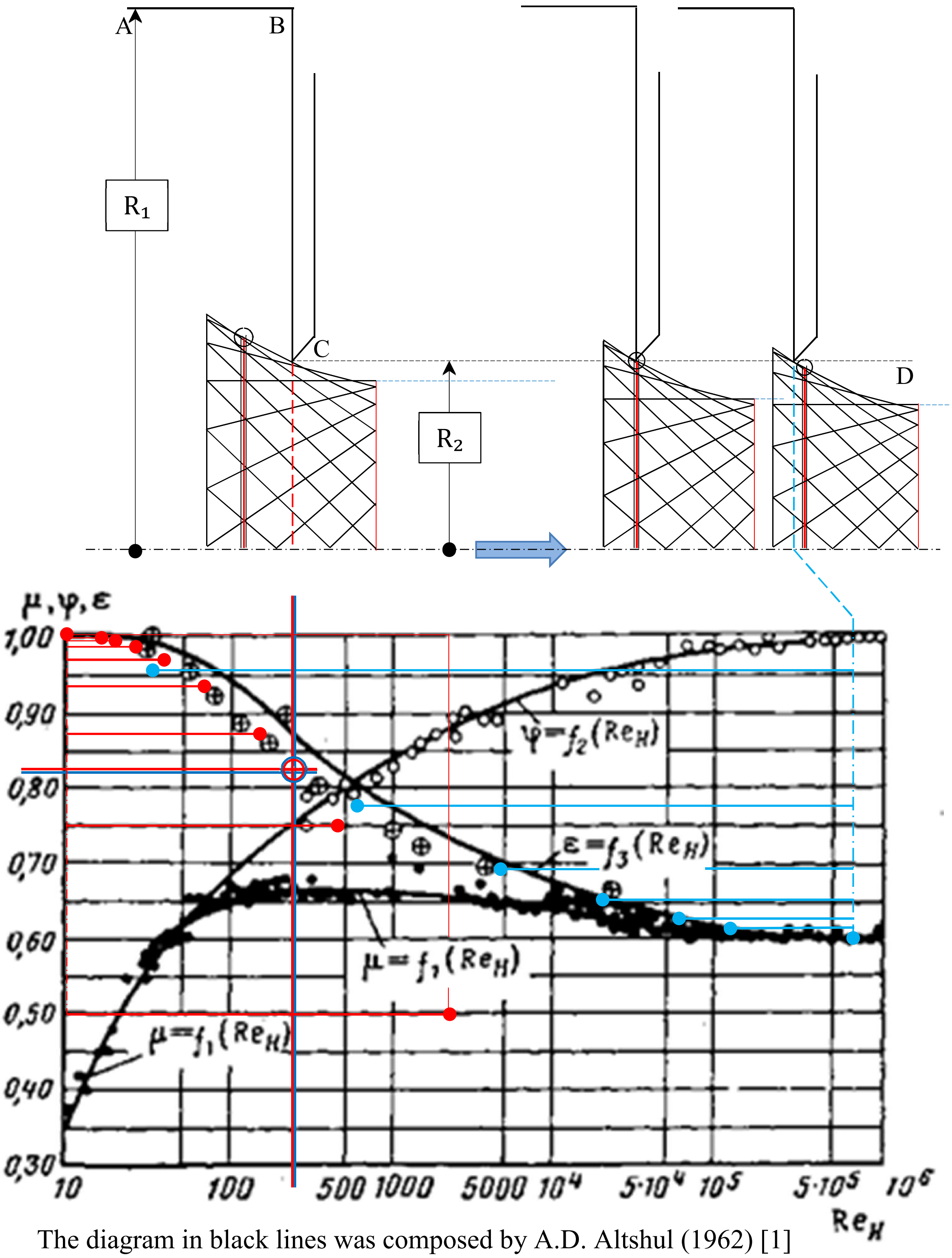


Fig.3



The diagram in black lines was composed by A.D. Altshul (1962) [1]

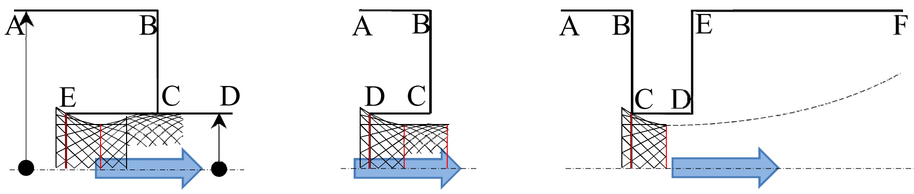
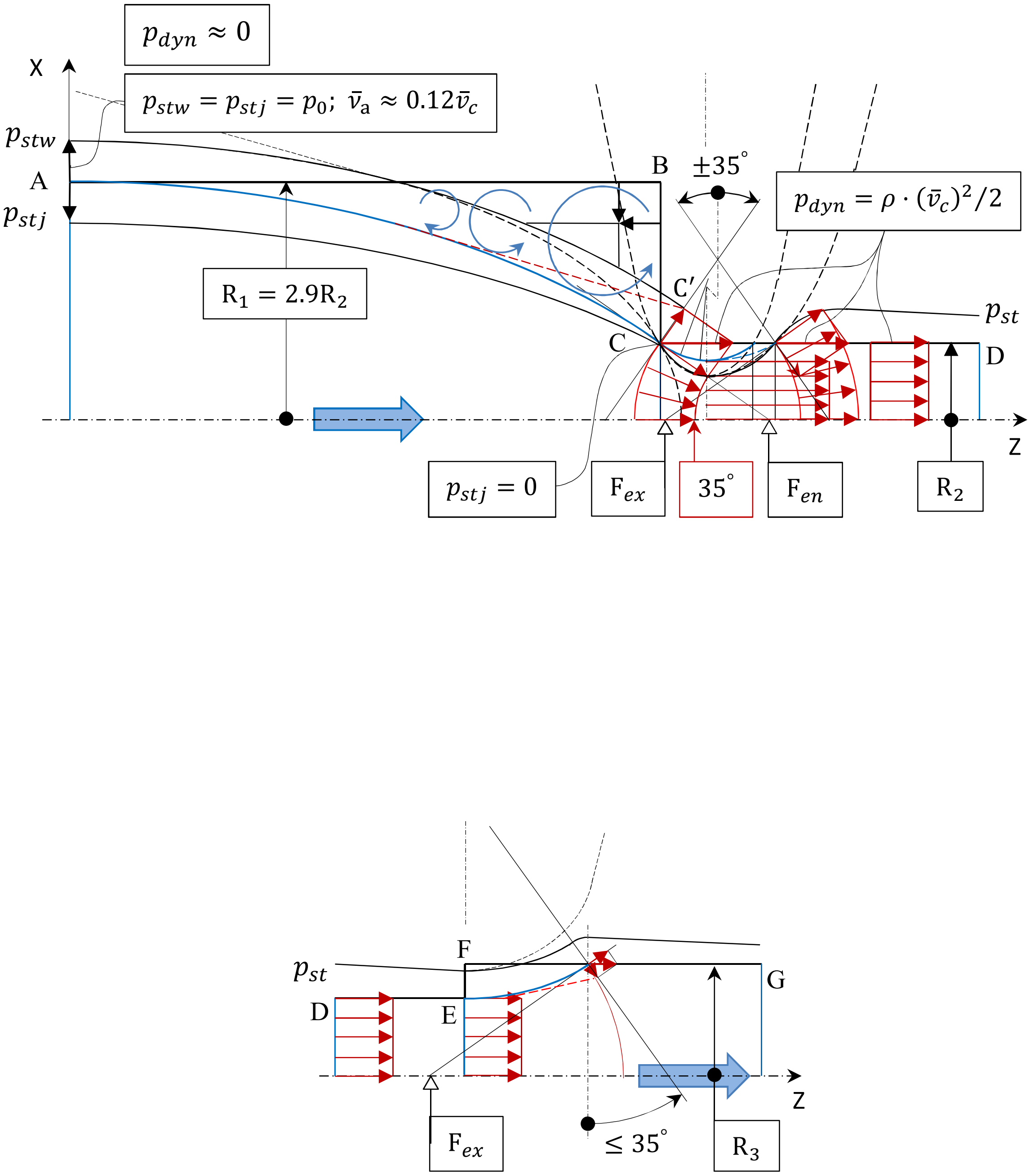


Fig.4



$p_{dyn} \approx 0$
X
$p_{stw} = p_{stj} = p_0;\ \bar{v}_a \approx 0.12\bar{v}_c$
p_{stw}
A
p_{stj}
B
$\pm 35^\circ$
$p_{dyn} = \rho \cdot (\bar{v}_c)^2/2$
C′
$R_1 = 2.9R_2$
C
p_{st}
D
Z
$p_{stj} = 0$
F_{ex}
35°
F_{en}
R_2
F
p_{st}
G
D
E
Z
F_{ex}
$\leq 35^\circ$
R_3

Fig.5

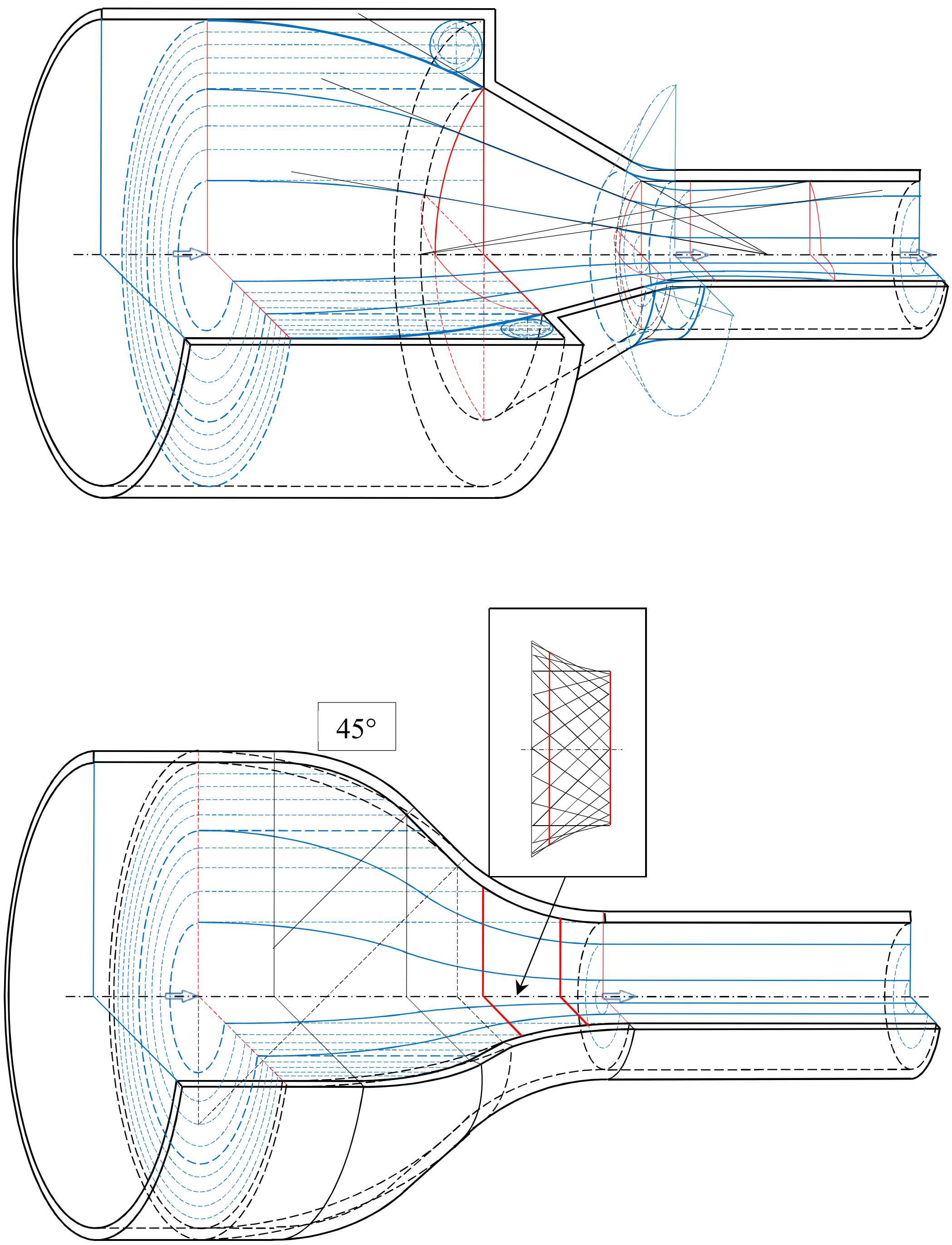


Fig.6

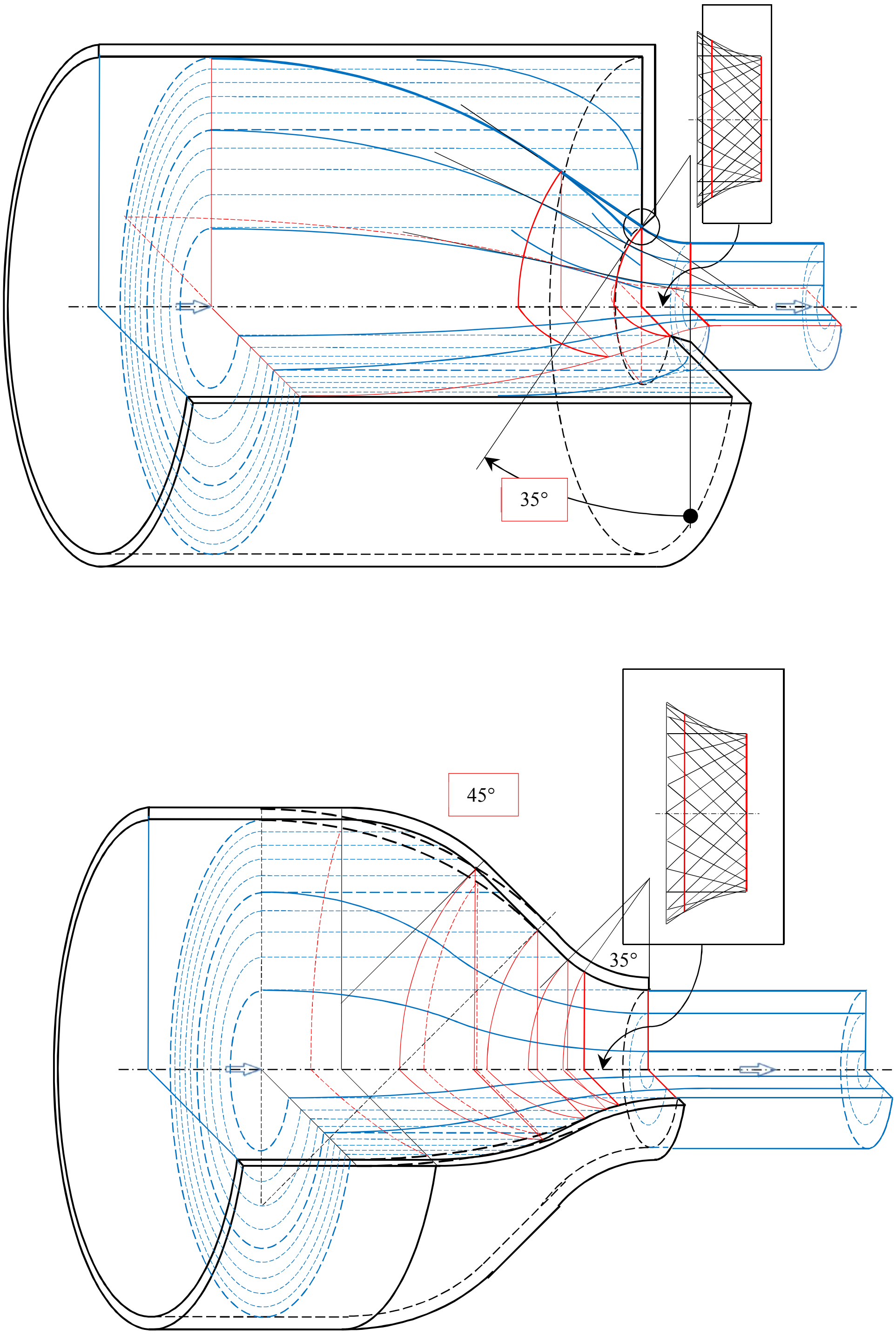
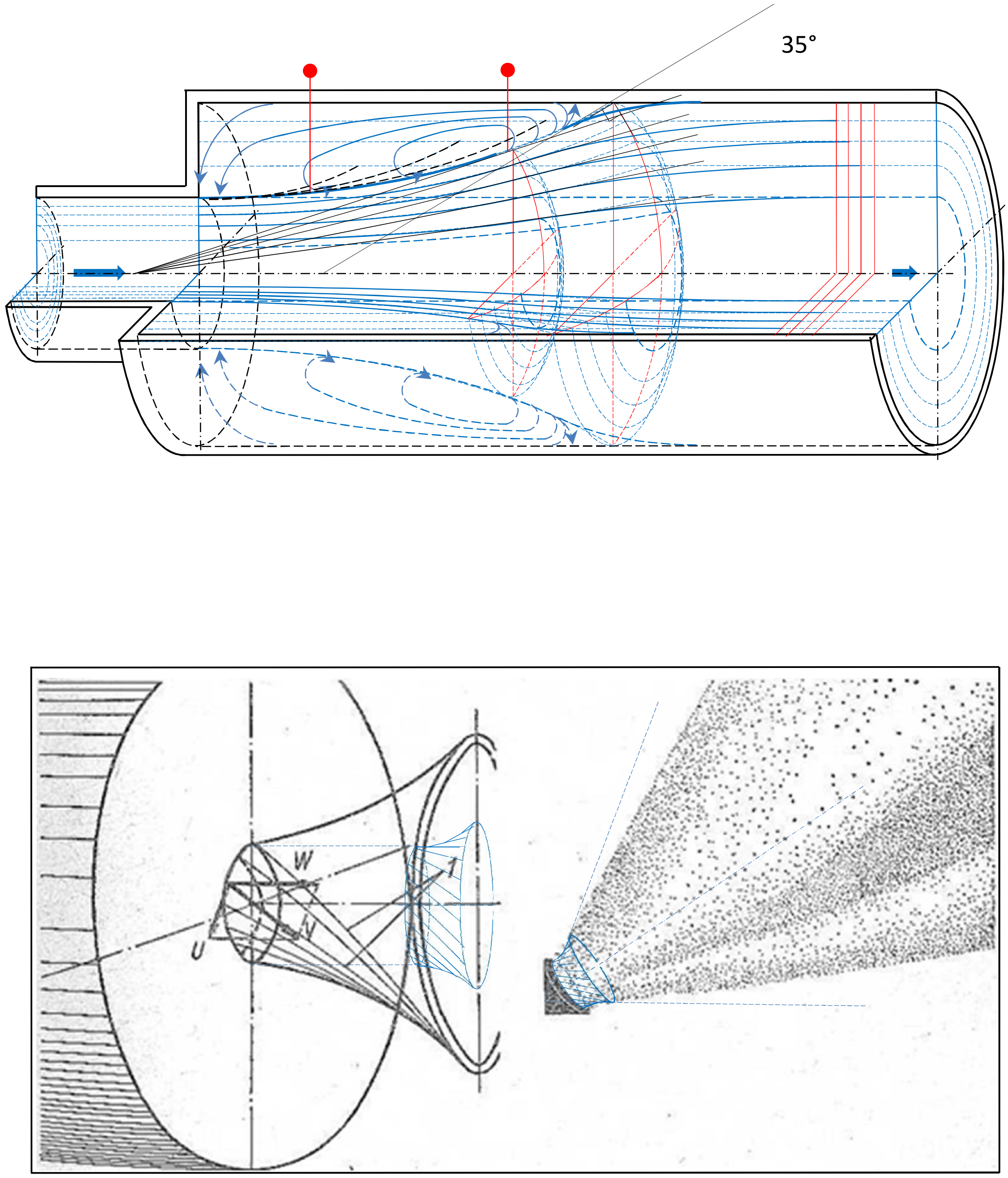
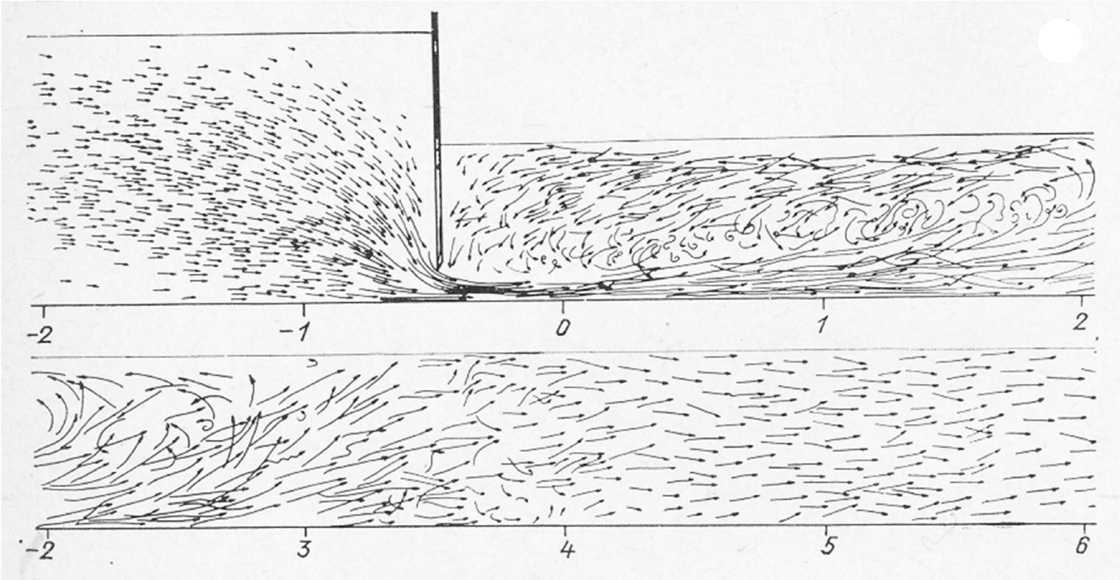


Fig.7



The diagram in black lines by M.S. Volynsky (1986) [2]

Fig.8



The field of velocities was visualized by M.S. Fomitchev (1967) [3]

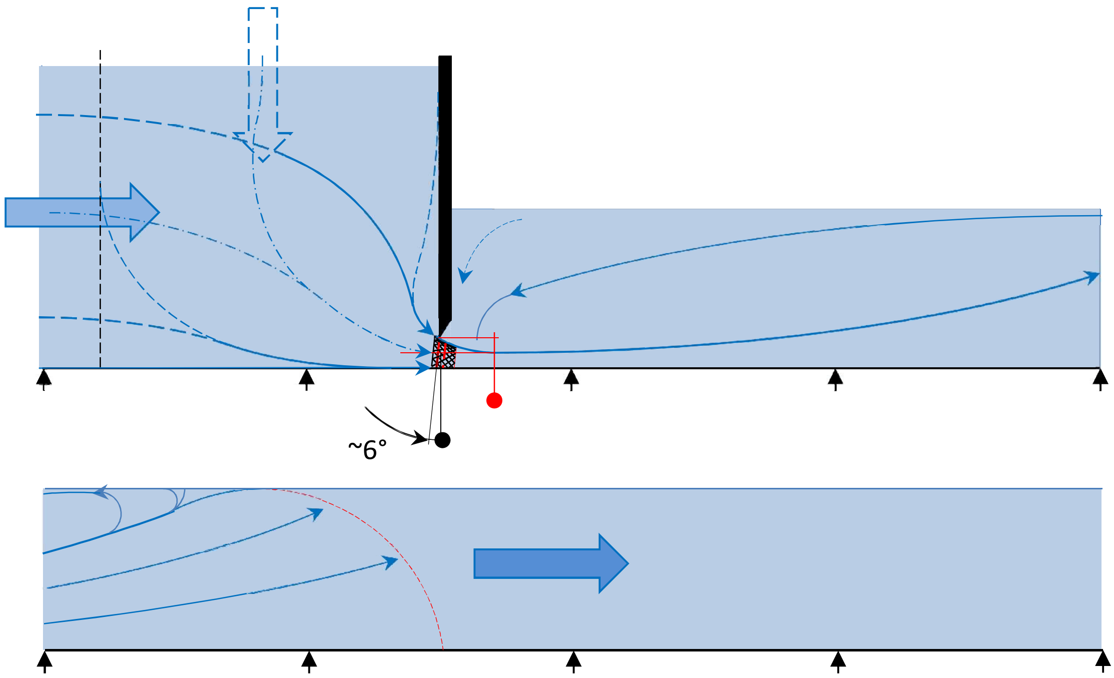
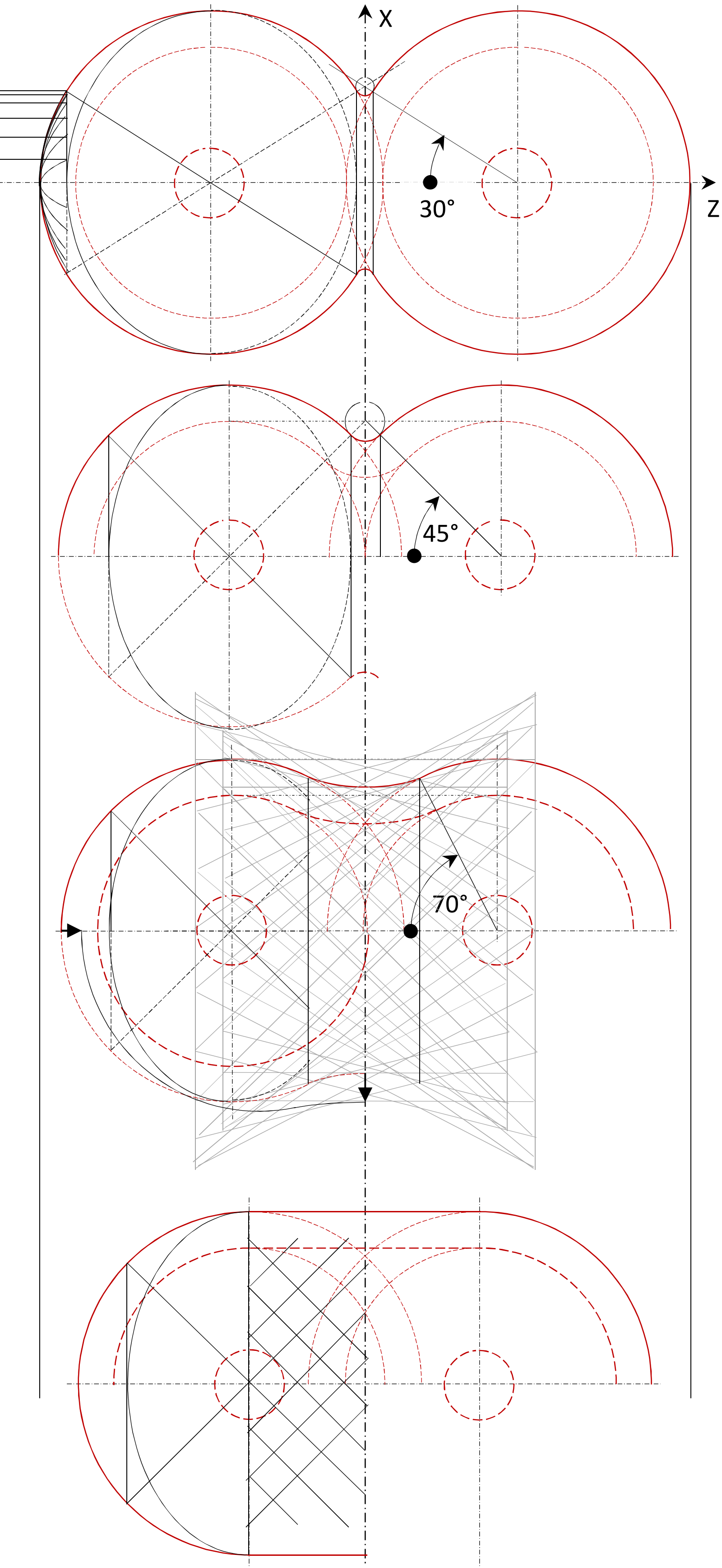
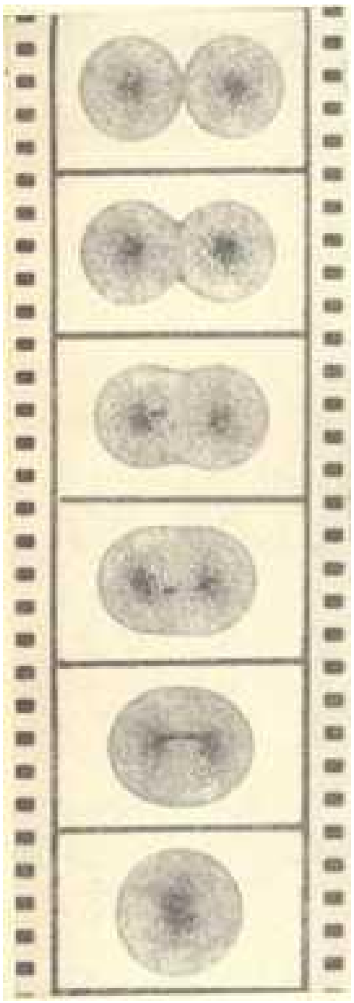


Fig.9





The cinefilm shots
by Ja.Je. Geguzin [4]

Fig.10

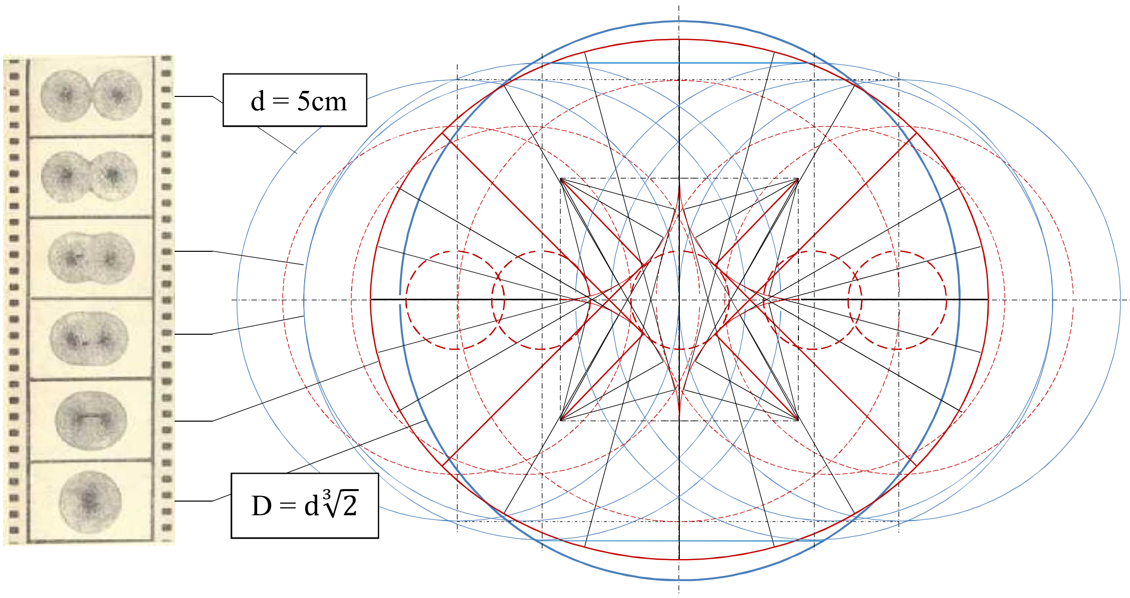


Fig.11