Mechanics of solid body, hydromechanics and gas dynamics in conic sections as a way for solution of the problems

Part 3: On mechanism of buckling of long bars and laminar motion stability of real fluid in tube

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Structural analysis of strained and stressed state in axisymmetric solid body, elaborated by the author in his previous articles, allowed to elucidate mechanism of buckling of long rods under longitudinal compression and now, with due regard for specificity of a liquid motion in tube, to produce mechanism of transition of laminar flow into a turbulent. Thereby the author corroborates a correctness of the transition established by him in one of his previous article under "Euler – Poiseuille criterion" name.
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Introduction

A question on mechanics of origin of turbulence in a fluid stream in straight cylindrical pipe is historically connected with problem on instability of laminar flow and one is considered as specific problems of hydromechanics at present. Experimental finding out of phenomenon in kind of transition of laminar flow of water in pipe to the turbulent is bound with names of G. Hagen (1839) and J. Poiseuille (1840), and the most careful experimental research of the question was carried out by O. Reynolds (1883).

In constructional engineering, actuality of problem on stability came into being in ancient times in view of necessity of stanchions as supporting elements for roof in housing and cult construction. In middle of XVIII century L. Euler deduced in the first time a formula determining the critical quantity of longitudinal force compressing a straight long rod. Euler supposed that the rod axis curvature after its buckling corresponds to a sinusoid one-half period, and quantity of compressive force is directly proportional to the rod bending rigidity and inversely proportional to square of its length. The formula went down in gold fund of mechanics side by side with the lever rules, but the phenomenon nature of the instability was found more complicated and requires subsequent experimental and theoretical researches in connection with diversity of the rod forms, the bearing constructions and also under dynamical longitudinal loads.

The author supposes to be rational to clear up of specificities of buckling as a process in elastic statement of problem, then in the elastic-plastic and only after that to consider a question on transition of laminar flow in straight pipe to the turbulent as the buckling process.

Approach

Experiments, carried out by English scientist E. Hodgkinson (1840), had corroborated correctness of Euler formula as applied to the rods with plane its ends. At the same time Hodgkinson had

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experimentally found out that for two flexible rods, one of which has plane its ends and the other was made with convex-rounded its ends, the second one was elastically bent by the same longitudinal force when its length was decreased in half in comparison with the first of them [1] According to Euler formula such experimental result signifies

$$P_{cr} = \left(\frac{\pi}{l}\right)^2 \mathrm{E}J_p = \left(\frac{2\pi}{l}\right)^2 EJ_r,$$

where p and r signify the plane and convex-rounded ends correspondingly.
Such unexpected result requires, in opinion of the given article author, elucidating of internal geometrical structure in the rod volume and also determining of rigidity in axial (longitudinal), hoop (transversal) directions and under bending in elements of the structure with taking into consideration of its interaction under longitudinal compressive force.
Thus, we are considering cylindrical rod with its diameter D = 1, made from medium-carbon steel and its length no less than L > 15D (i.e. λ > 100).
Conception of internal geometrical structure in cylindrical rod, introduced by the author, includes on initial stage in itself: a rod core, a rod shell and near-axial singular cylinder with its uncertainly small diameter.

Fig.1, in its left side, shows consecutive division of the cylindrical rod cross-section area in half when the rod is loaded by longitudinal compressive force; at the same place there are showed helical trajectories 45° of one – right-handed - of two sets of principal stresses; the right side shows the same division of the same rod under action of torsion moment: in this case is added a boundary corresponding to equality of polar moments of inertia of the rod shell and core parts Diameter of the core is equaled $d_c = D/\sqrt{2}$. Correspondingly to it the cross-section areas of core and shell equal to each other and thereby ensure equality in its axial rigidity.
Moment of inertia under bending of the core

$$J_c = \pi\left(D/\sqrt{2}\right)^4/64 = 0.25\pi D^4/64,$$

and moment of inertia of the shell

$$J_S = \pi D^4\left[1-(d_c/D)^4\right]/64 = 0.75\pi D^4/64,$$

i.e.

$$J_S/J_c = 3/1.$$

So, it should be supposed that the shell geometrical structure consists of three annular layers having the same quantity of its moment of inertia under bending equal $0.25\pi D^4/64$.
Moment of inertia of the shell external layer

$$J_{Se} = \pi D^4\left[1-0.9306^4\right]/64 = 0.25\pi D^4/64;$$

moment of inertia of the shell middle layer

$$J_{Sm} = \pi D^4\left[1-0.841^4\right]/64 - 0.25\pi D^4/64 = 0.25\pi D^4/64.$$

Thus, full internal geometrical structure of considered cylindrical rod contains in itself:
- core with its diameter $d_c = 0.7071D$;
- shell consisting of three annular layers: $d_c/d_{in} = 0.7071/0.841$;
$d_{in}/d_m = 0.841/0.9306$;
$d_m/D = 0.9306/1.000$;
- near-axial singular cylinder.

Thickness and relative hoop rigidity of the shell annular layers

$$\delta r_{in} = (0.841 - 0.7071)\cdot D/2 = \ 0.067D \sim 1.00;$$
$$\delta r_{m} = (0.9306 - 0.841)\cdot D/2 = \ 0.045D \sim 0.67;$$
$$\delta r_{ex} = (1.0000 - 0.9306)\cdot D/2 = 0.035D \sim 0.52.$$

Relative area of cross-section of these layers and correspondingly its axial rigidity

$$a_c \sim 0.7071^2 \sim \quad 0.5A_D \sim \quad 1.0;$$
$$a_{in} \sim (0.841^2 - 0.7071^2) \sim 0.207A_D \sim 0.414;$$
$$a_m \sim (0.9306^2 - 0.841^2) \sim 0.159A_D \sim 0.318;$$
$$a_{ex} \sim (1.000^2 - 0.9306^2) \sim 0.134A_D \sim 0.268.$$

Thus a using of notion on internal geometrical structure in volume of cylindrical rod from polycrystallic statistically isotropic material has allowed elucidating distribution of bending, axial and hoop rigidity as factors of three-dimensional resistance of straight rod to longitudinal compressive load.

The three-dimensional state of the rod with a ball and socket bearing of its ends is stipulated by field of internal forces in kind of two orthogonal each other sets of helical lines 45° - dextrorse and sinistrorse – as trajectories of principal stresses.

Strictly symmetrical realization of the experiment not allows foretelling in what side the bending will be occurred; but we know at advance that the bending will be flat; we know also that the bent axis of the rod will be follow half-period sinusoid; and now we know that the flat line is a profile of half-coil of the helical lines as trajectories of the field of internal forces, determining existence of phenomenon under name buckling. The force field stipulates existence on the rod convex side tensile stresses not only in longitudinal direction but also in transversal – hoop direction; correspondingly on the rod concave side compressive stresses are acting not only in longitudinal direction but also in hoop direction. Symmetricity of action of the hoop stresses relative to the rod bending plane stipulates existence in every of two halves its cross-section a moment of torsional forces – equal in its quantity and directed in opposition sides: from the rod convex side to its concave side, i.e. pair of moments reflected by the rod bending plane as mirror.

Using the above-described approach, we can for example compare stability of two the same tubular rods at two variants of its loading:

- longitudinal force is immediately applied to the rod flat ends;
- longitudinal force is applied to two pistons compressing fluid in the rod cavity at lack of friction.

We suppose a ball and socket joint as a bearing for the first rod ends and for the pistons of the second rod. We also suppose a ratio of diameters $d_c = D/\sqrt{2}$ of these rods that ensures equality of the cross-section areas of the rod shell and cavity in it.

In the first of these two variants, an answer is obvious: a quantity of critical force for such tubular rod is three quarters of that quantity for the same but solid rod.

In the second variant, tubular rod is a heavy-walled cylinder loaded with internal pressure of a fluid. G. Lame's (1833) formulas determine hoop tensile stresses on internal surface of the cylinder

$$\sigma_t^d = \frac{1 + A_c/A_D}{1 - A_c/A_d}p = \frac{1 + 0.5}{1 - 0.5}p = 3p$$

and on its external surface

$$\sigma_t^D = \frac{2 \cdot A_c/A_D}{1 - A_c/A_d} p = \frac{2 \cdot 0.5}{1 - 0.5} p = 2p.$$

Relative decrease in hoop tensile stresses is 0.67. At the same time the above-determined relative decrease in hoop rigidity of the rod shell layers is 1; 0.67; 0.52. In that way these hoop tensile stresses overcome elastic resistance of the shell annular layers more than three times and ensure, with taking into consideration of Poisson's coefficient μ = 0.3, axial elastic shortening of the tubular rod, corresponding to its buckling by Euler.
Quantity of relative elastic shortening of flexible cylindrical rod with compact cross-section, corresponding to critical force by Euler,

$$\varepsilon_{cr} = \frac{P_{cr}}{EA} = \frac{\pi^2 EJ}{L^2 EA} = \left(\frac{\pi}{4}\right)^2 \left(\frac{D}{L}\right)^2$$

is decreased by hyperbolic dependence as the rod relative length, expressed by it diameter, is increased; maximum quantity as applied to medium-carbon steel is

$$\varepsilon_{cr}^{max} = \left(\frac{\pi}{4}\right)^2 \left(\frac{1}{12.5 \ldots 15}\right)^2 = 0.0039 \div 0.0027.$$

Quantity of relative elastic shortening of flexible tubular rod with $d_c = D/\sqrt{2}$, corresponding to critical force by Euler,

$$\varepsilon_{cr} = \frac{\pi^2 EJ}{L^2 EA_{dD}} = \frac{3}{2} \left(\frac{\pi}{4}\right)^2 \left(\frac{D}{L}\right)^2.$$

On the whole numerous well-knowing experimental and theoretical researches and adduced here example of three-dimensional analysis of rod stressed state are evidence to that the Euler's formula is correct as applied to flexible massive and tubular rods with flat ends.
At the same time, in order to elucidate a cause of Hodgkinson's unexpected result, on the author's view, it should be considered the buckling phenomenon as a physical process concluding in it subsequently stages of latent origin, development and suddenly observed result in kind of bending of straight rod under longitudinal compressive force. Especial role of near-axial singular cylinder in the process is stipulated by a number of factors:
- just the cylinder immediately undergoes action of external longitudinal compressive force;
- diameter and area of contact of the cylinder with external longitudinal compressive force are neglected small in comparison with the rod cross-section diameter;
- as element of internal geometrical structure, the cylinder has indeterminately-small area of its cross-section and correspondingly indeterminately-small its moment of inertia under bending;
- the cylinder is called by a singular since one can increase its flexibility up to endlessness when radius of inertia of it cross-section is decreased up to zero.

Combination of these factors results to that an action of compressive force onto a rod with convex-rounded ends is accompanied by consecutive passing ahead in compression of central – singular – cylinder, then in compression of the rod core: from central part to periphery of its cross-section, and after that the compression, consequently decreasing, spreads onto layers of the rod shell part. Role of singular cylinder as an exciter of the rod buckling is in that one itself and adjoined to it thin layers of the rod core have been found in a state corresponding to its buckling under action of already not great axial compressive load. Axisymmetricity of such state is held out by bending rigidity of the rod shell layers. This state of possible balance of the rod can be disturbed by slightest asymmetricity, and then problem on longitudinal bending gives place to problem on cross bending: very small lateral force in middle of the rod length creates a bending moment with arm equal half length of the rod. A quantity of the bending moment is quite sufficiently for visible bending of the rod and for subsequent return it in initial straight state after taking down of longitudinal compressive force. Just in that very case a quantity of longitudinal compressive force, applied to the rod, equal a quarter of force which one is necessary for buckling of the same rod with flat ends. It is precisely this fact that explains that is why Hodgkinson had to halve the rod length. In contrast to it a loading of cylindrical rod with flat ends is going on synchronously in both the rod core and shell as well central singular cylinder, and resistance of the rod to longitudinal compressive force is determined by summary axial rigidity of all elements of the rod cross-section in accordance with Euler's statement of the problem.

Fig.2 shows three flexible cylindrical rods of the same length loaded with longitudinal compressive force: Euler's critical force for the left rod is a sum of the same four forces correspondingly to summary bending rigidity of the rod core and three layers of its shell; Euler's critical force for middle rod is a quarter as its cross-section is restricted by the rod core; a buckling of the right rod is also going on at a quarter of Euler's force because of its convexo-rounded ends. Specific influence of the singular cylinder onto the rod stressed state and its load-carrying ability we found also in tension of cylindrical specimen from medium-carbon steel: in this case an initial transversal crack appears just in very central part of its cross-section; maximum quantity of radial compression in cylindrical rod under torsion moment is reached at the rod axis, and forced elasto-plastic elongation of singular cylinder and the rod core together with it, stipulated by the radial compression, arouses a loading of the rod shell with longitudinal tension up to its fracture. Thus our method of the stressed and strained state analysis not allows finding out positive influence of central singular cylinder and adjoined to it layers of the rod core onto its load-carrying ability under axial compression, tension and torsion. We can corroborate physical adequacy of such conclusion by means of comparison with similar elements in Nature's engineering:

- stem of herbs has layered fibrous hollow structure; stem of rye, wild cane, bamboo has $d/D = 0.9$, $L/D = 150$ and more; internal cavity of herbs can be filled with nutrient solution;
- trunk of trees has annular layered grained structure with consecutive diminution in a wood density to the cross-section center;
- birds: crane, heron, flamingo has tubular legs $d/D = 0.77$, $L/D = 20$;
- the man and animals skeleton contains two kinds of bones: the tubular $d/D = 0.65$, $L/D = 8$, partially filled with oriented or isotropic osseous latticed or porous structure, and sandwich shells consisting of two thin-walled shells and porous structure in closed space between its.

Fig.3 offers a photo of a shin-bone in longitudinal (vertical) section: we can see tubular middle part of the bone and two its end parts filled with isotropic osseous porous structure; the photo is adopted from An Atlas of Man Anatomy [2].

Fig.4 offers another photo, from the same book, of a thigh-bone in hip articulation in vertical section: we can see tubular part of the bone and its end part filled with oriented osseous latticed structure; at the same place we can also see a pelvic bone in kind of sandwich shell filled with isotropic osseous porous structure.
Absolute lack of central singular cylinder in adduced examples should be considered as one of principle in Nature's engineering which one not only corroborates above our conclusion, but also determines hollow rods, shafts, axles, columns, springs as rational elements of machines and mechanisms as well building constructions having both high short-term strength and durability under action of axial tensioning and compressive forces and also torsion moment.

Only now we have necessary physically adequate prerequisites for reconsidering of a question on the laminar flow instability in straight cylindrical pipe, solved by the author in one of his previous articles [3].
Three following theses:
- volume of statistically-isotropic solid rod with cross-section in kind of circle or regular polygon and fluid stream in straight pipe with the same cross-section contains in both these cases internal layered-annular structure, determined by its axial, bending and torsion rigidity;
- principal stresses in the solid rod and principal motive forces in the fluid stream are acting along trajectories of two sets of helical 45° lines; balanced in its quantity, these forces ensure forward motion of the stream in pipe;
- curves from conic sections are base for graphical-analytical solution of problems on a solid rod deformation and a fluid stream motion in pipe;
in combination with fundamental heritage, obtained by penetrating mind of our precursors, open a possibility for physically adequate and mathematically correct statement and solution of problems in mechanics of solid body and fluid medium.

Fig.5 shows that the stationary laminar water stream in straight cylindrical pipe can be presented in kind of a rod with its length L, diameter D and its core $d_c = D/\sqrt{2}$; the rod being under action of axial and radial compressive forces stipulated by static pressure, and at the same time one is moved rectilinearly and uniformly, i.e. with constant velocity similarly to inertial movement.

Fig.6, in its upper part, offers classical distribution of the friction forces and velocity along the pipe radius in the laminar stream and at the same time one shows that paraboloid of rotation, formed by parabola in second power, divides a volume of circumscribed around it cylinder in radial and longitudinal directions onto four parts equal each other as the stream cross-section areas of its core and shell parts as well as two halves along the stream axis are equal each other. This is geometrical explanation to that the mean (in flow rate) quantity of the stream velocity is half maximum its velocity (in the stream axis).
The lower diagram shows rectilinear motion of the stream in straight pipe as a result of action of a field of motive forces directed along trajectories of two sets of helical lines inclined under 45° to the stream axis. In straight pipe, these forces balance each other and ones together ensure forward movement only, and correspondingly to it we can write expression for axial velocity head

$$p_{dyn} = \rho \frac{\bar{v}^2}{2} = \frac{\rho}{2}[(\bar{v}_{lh} \sin 45°)^2 + (\bar{v}_{rh} \cos 45°)^2],$$

where $\bar{v}_{lh} = \bar{v}_{rh}$ are velocities along the left- and right-handed helical trajectories correspondingly. But in the bent pipe, these two sets of helical trajectories are separated against each other with a forming of pair-helical (pair-spiral) flow; such phenomenon quite similar to above-described stressed state of bent solid cylindrical rod at its buckling. At the same place, it is showed two cylindrical boundaries: one of these $d_c = D/\sqrt{2}$ divides the stream cross-section area in half and the other $d_{cp} = D/\sqrt[4]{2}$ ensures equality in polar moments of inertia of the stream cross-section core and shell parts; as applied to well-known “bath effect” and similar phenomena, a thickness of the stream circular shell, determined by the second boundary, determines, in one’s turn, quantity of revolving jets which ones form rotatory movement of water inflowing to a bath outlet. Near-axial singular cylinder with its uncertainly small diameter is also showed in both diagrams of the fig., since Reynolds in one of his remarkable experiments not only has distinctly found out such element of a water stream in cylindrical tube, but he had also disclosed role of the flow element as an exciter of transition of the water flow from laminar to turbulent form.

Fig.7 offers three examples of rectilinear plano-symmetrical water flow: along horizontal flat plate, in open straight channel and in straight pipe with partially filled its cross-section. Feature in the first of these examples is in that the flow is the same both - upper and lower - surfaces of the plate. General property of all three these examples is in that decrease of velocity head in axial direction, caused by friction, arouses equal to it by quantity pair of transversal velocity heads, directed in both sides away from the plane of symmetry; using Darcy-Weisbach formula, we obtain

$$\rho \frac{v_y^2}{2} = \lambda \frac{l}{d} \cdot \rho \frac{v_x^2}{2},$$

i.e. loss of velocity head in forward flow is compensated by the transversal velocity head on both sides from the plane of symmetry. This pair of transversal velocity heads flattens water jet with its initial circular cross-section on flat surface of the plate, and one forms pair-spiral flow in above open channel and the pipe. Straight pipe full filled with water and with its cross-section in kind of equilateral triangle has three planes of symmetry passing through its apices, and therefore water stream in the pipe contains three pair-spiral components in its forward motion.

Fig.8 offers conditionally a fragment of the water stream in a pipe with circular cross-section. Longitudinal section of the fragment contains, in squared frame, two parabolas: in the second power (black line) and in the fourth power (red line), constructed by pieces of straight line; the first of these parabolas is a profile of the stream velocity in its laminar flow, and the second parabola is a profile of the stream velocity head; in other words, the velocity profile is a function of the stream radius in second power, and the velocity head profile is a function of the stream radius in fourth power. Static pressure is showed in kind of spheres with its same radius, placed along the stream cross-section radius. It should be noted, that a coordinate system for the stream is set by the pipe internal cylindrical surface. In the system, maximum quantity of a friction force at the pipe internal surface is decreased uniformly up to zero at the stream axis, i.e. where is placed singular cylinder; the cylinder closes by itself the stream cross-section area and transforms the friction resistance into static pressure, directed radially from the stream axis to the pipe wall, and one is remained by equal in both radial and axial directions in the given cross-section of the stream. In contrast to it, the velocity head is remained by equal along the stream forward flow, but one is decreased in radial direction from maximum quantity at the stream axis to zero at the pipe wall in the given section.
The simplest device of H. Pitot (1732) is the invention of genius, since the device has opened a possibility of measuring of both the so called full velocity head, as a sum of axial velocity head and static pressure, and simultaneously static pressure separately. Since this sum is vectorial, we must add geometrically the same static pressure, acting also and in radial direction, to the sum; just namely such sum determines physically adequately the stream full head vector with its components in kind of the summary axial velocity and static head vector and also radial, static, head vector. In its nature, as foreign body in fluid stream, Pitot device cannot measure the stream full head vector.
The water stream in pipe is not only geometrically restricted by cylindrical wall, but also, in its forward motion, one interacts with the wall, and therefore one is pierced through by axisymmetric field of the lengthwise interlayered forces of the fluid internal friction; these two factors stipulate laminar flow of the stream.
Reverting to diagram in the figure, we see that, in contrast to the friction forces, the full head vector in singular cylinder is deflected from the stream axis, and one acts along the generating line of conic surface; this deflection angle is increased correspondingly to a decrease of velocity head, as axial component of the full head vector, in radial direction to the pipe wall. Forces of the full head vector are forming axisymmetric field, piercing through the water stream.

One of these two, a field of the friction forces, is a primary factor determining motion of a fluid stream as such. The field determines completely existence of laminar flow at comparatively low velocity, and one can, in perfect experiment, remain at very high velocities of a stream flow in pipe. A field of the full head forces is secondary factor, as a consequence of a fluid motion, formed by the friction forces. The field is very intensively increasing its energy as a stream velocity is increased Both these fields – of a friction and full head forces – are forming and operating the stream flow in pipe, and at the same time themselves remain immovable relative to every section of pipe. A state of rectilinear and uniform steady motion of stream is the state of relative motion; such state is equivalent to immobility in itself. Pressure drop, applied to the pipe ends, is axial force compressing the stream in kind of sufficiently long cylindrical column. Under the circumstances, just namely the field of the full head forces, which possesses:

- intensity proportional to the fourth power of the stream radius;
- central singular cylinder as exciter, very sensitive to slightest imperfections in the flow system;

determines a transition from laminar to turbulent flow as phenomenon of buckling in sufficiently long cylindrical rod above-described.
By his experiment, O. Reynolds has showed an origin of turbulence in kind of transition of central tinctured jet to its motion along helical line on surface of invisible cone.

Fig.9 shows, in black lines, diagram by J. Nikuradse (1932), adopted from Schlichting's book [4]. The diagram offers six profiles of velocity in a turbulent water flow through a smooth-walled cylindrical pipe, which ones correspond to the power dependence with its power in range from the sixth to the tenth power of the stream radius. At the same place, author of the given article has adduced a velocity profile of laminar stream in kind of parabola in the second power. A mean velocity, determined by a flow rate, in laminar stream corresponds to the pipe radius:
$r_l = R/\sqrt{2} = 0.707R$, i.e. on distance from the pipe wall 0.293*R*; and the mean velocity of turbulent stream corresponds to the pipe radius

$$r_t = \left(R/\sqrt{2} + R/\sqrt[4]{2}\right)/2 = 0.774R,$$

i.e. on distance from the pipe wall 0.226*R*; in other words, the mean velocity radius of turbulent stream equal exactly to a half sum of radii dividing the stream cross-section into two parts: with equal quantity of its area and also with equal quantity of its polar moments of inertia. A quantity of a mean velocity of turbulent stream exceeds that quantity of laminar stream: $\bar{v}_t/\bar{v}_l = 1.75$. The author supposes it to be rational to call the laminar flow in the limits up to Reynolds number 2000…2300 by ordinary laminar flow and the same flow behind these limits to call by a superlaminar, since such flow can be realized under special conditions only.

Fig.10 shows, in black line, a parabola in the second power – in the limits y = x^2 = 1 – as profile of velocity of water stream in cylindrical pipe, corresponding to ordinary developed laminar flow; the same parabola, symmetrically reflected by diagonal of square, is found in a number of parabolas, which ones determine profiles of velocity in developed turbulent stream at its power from 6 to 10 in exact conformity with results of J. Nikuradse's experiments [4, fig.20.2]. In a case of superlaminar flow, its velocity profile turns out elongated y = $x^2 \gg 1$, as it is showed in fig.253 in [6].

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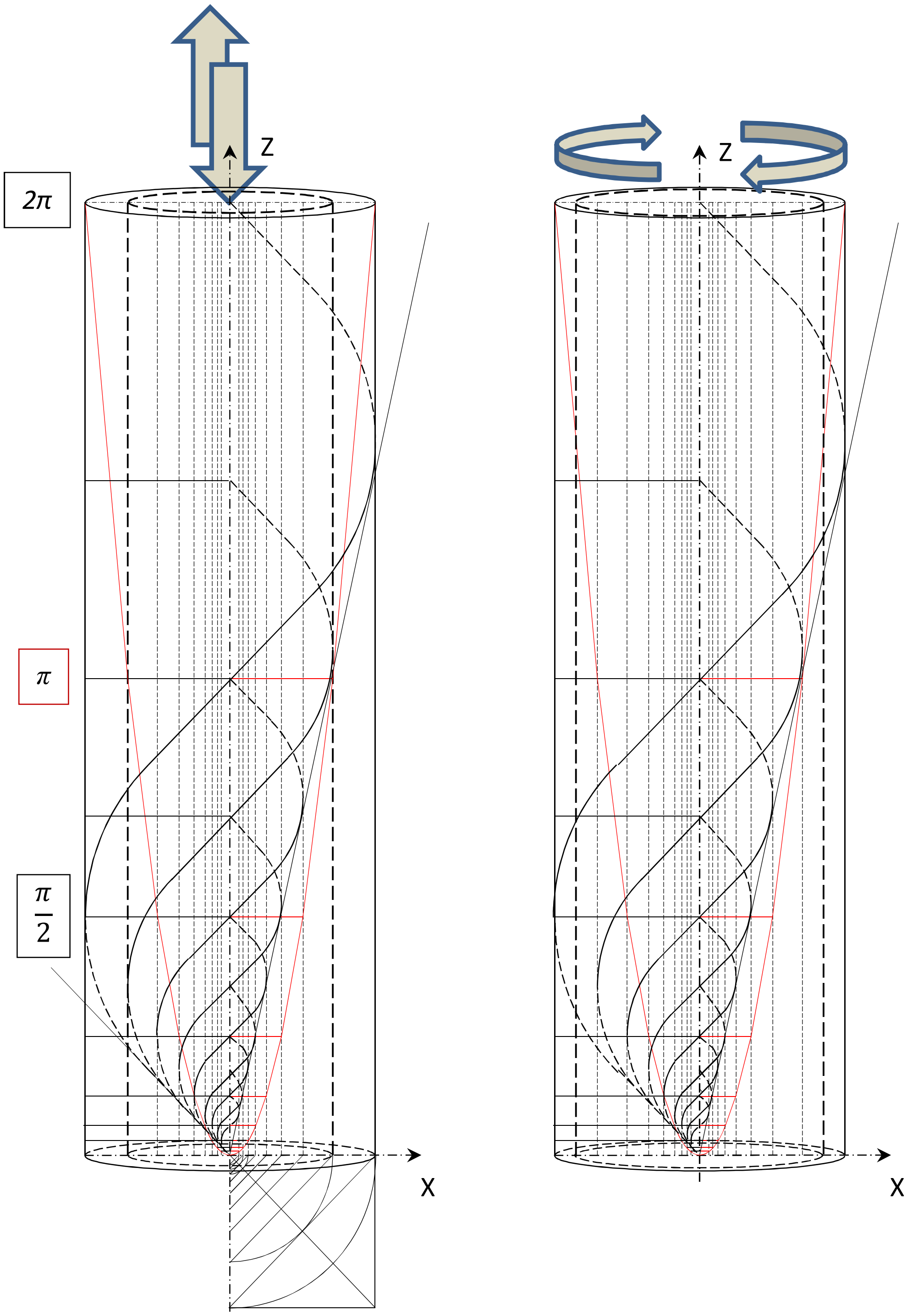


Fig.1

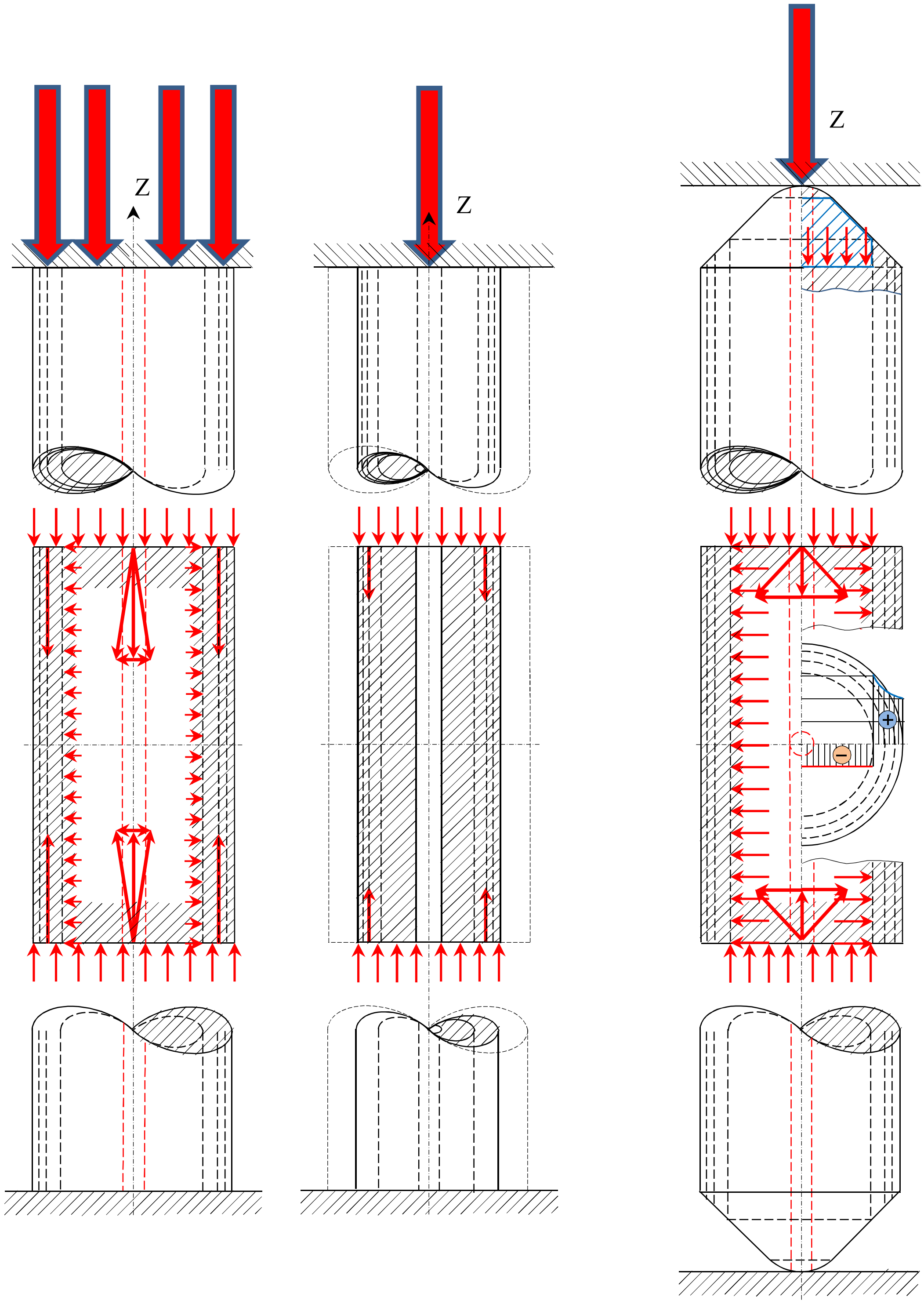


Fig.2

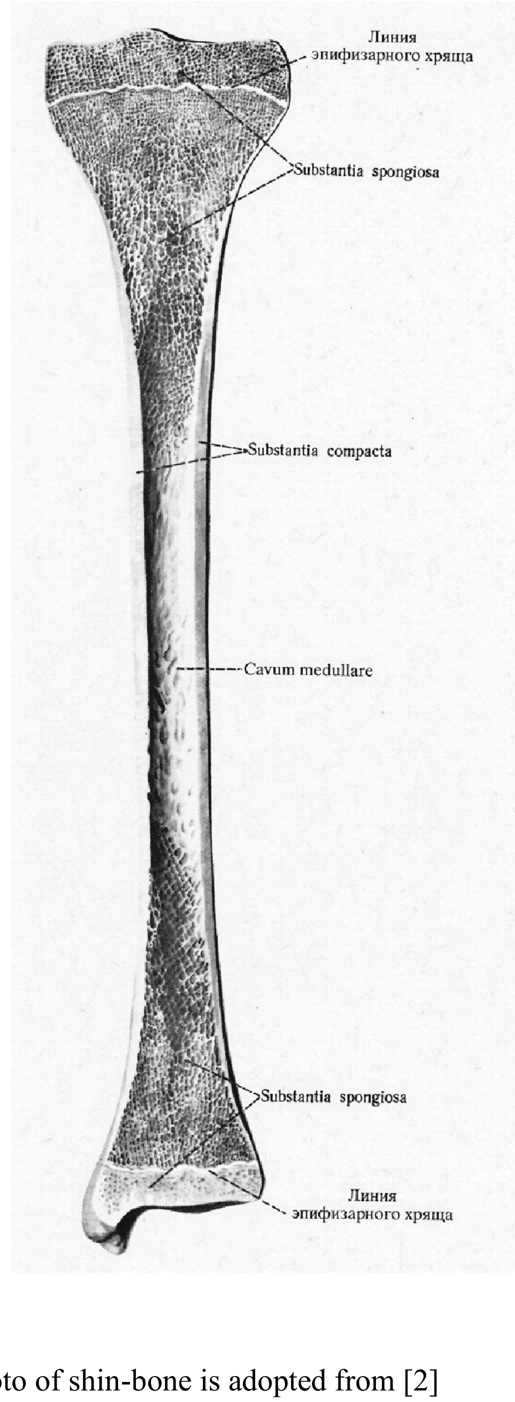


Fig.3: the photo of shin-bone is adopted from [2]
(Longitudinal cutting)

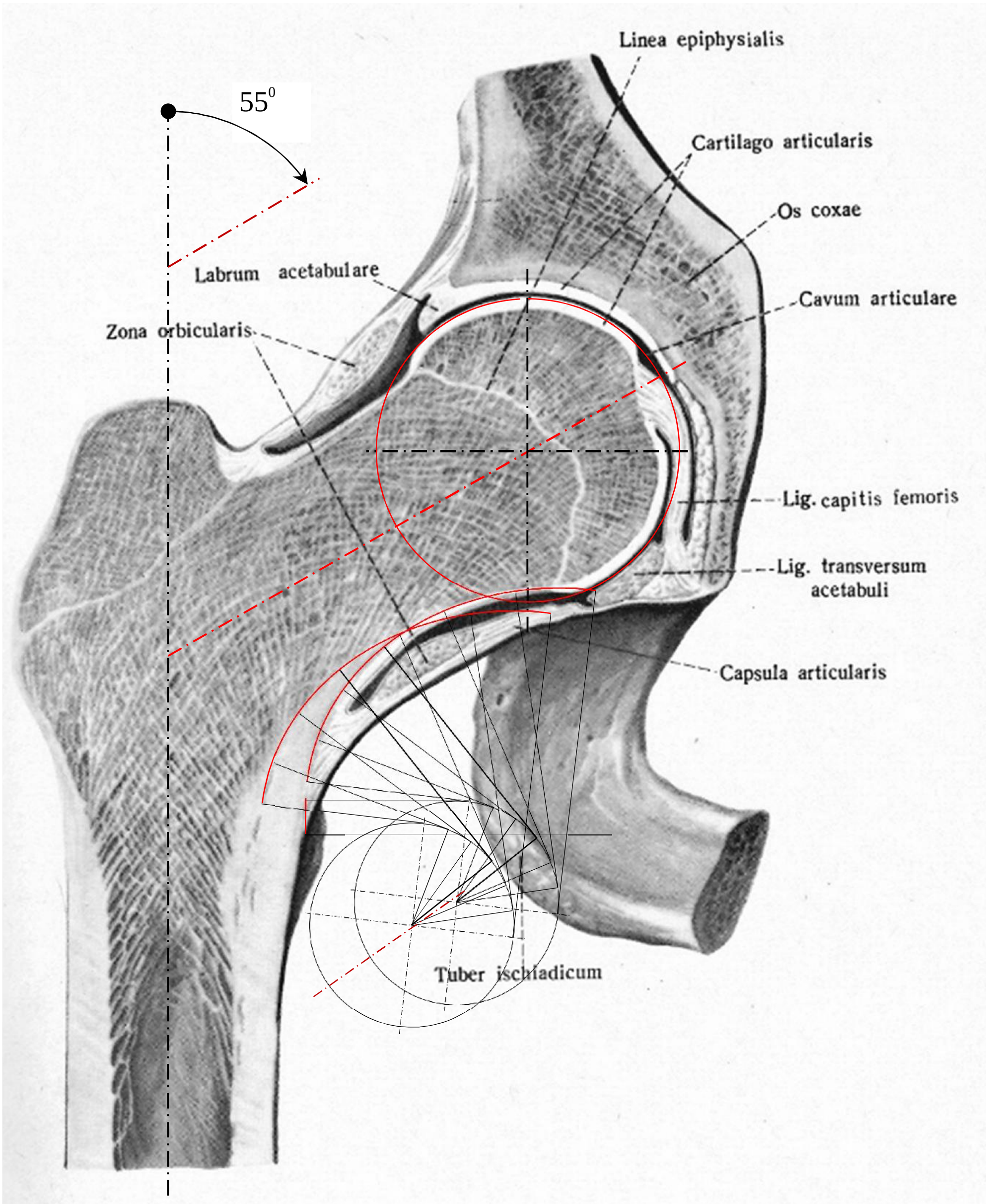


Fig.4: the photo of hip articulation is adopted from [2]

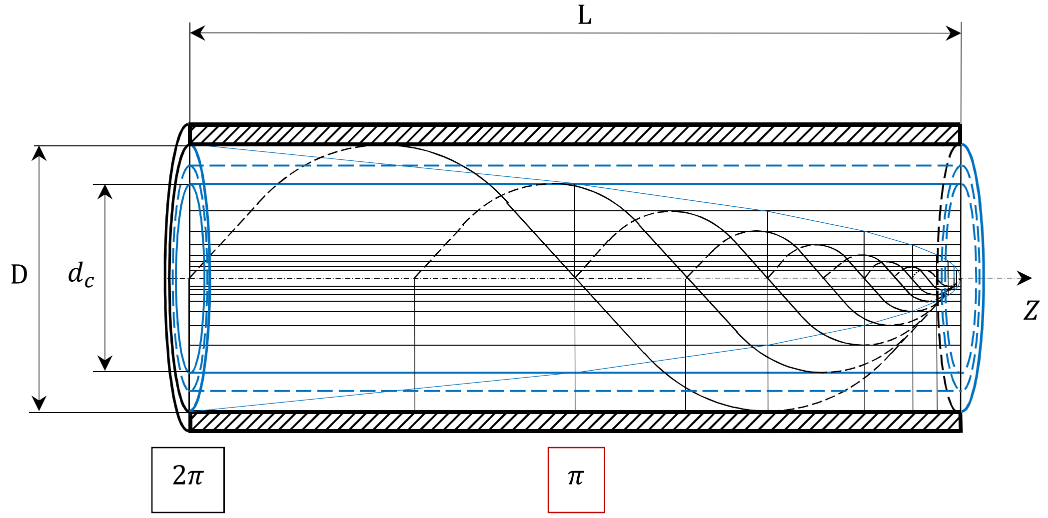


Fig.5

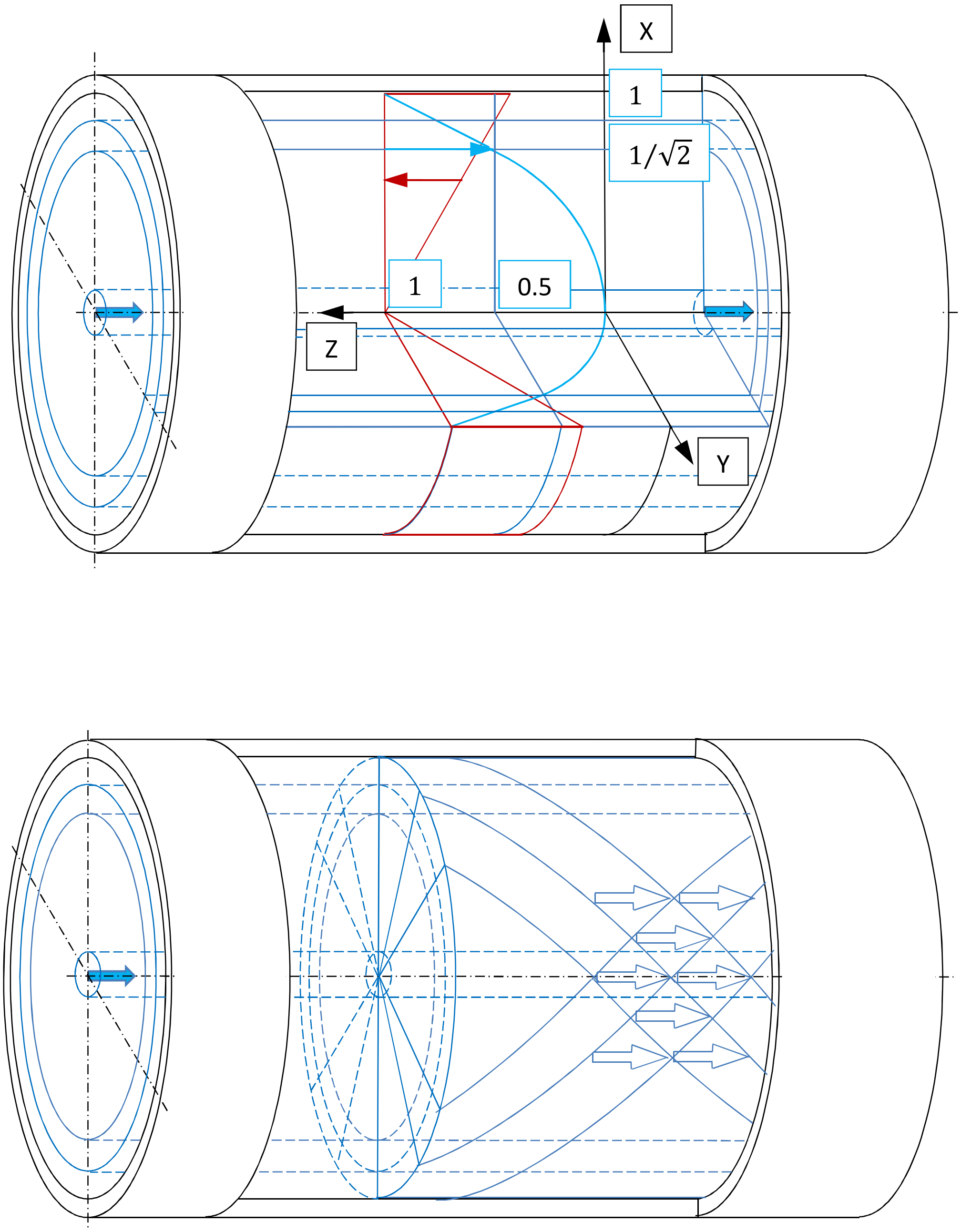


Fig.6

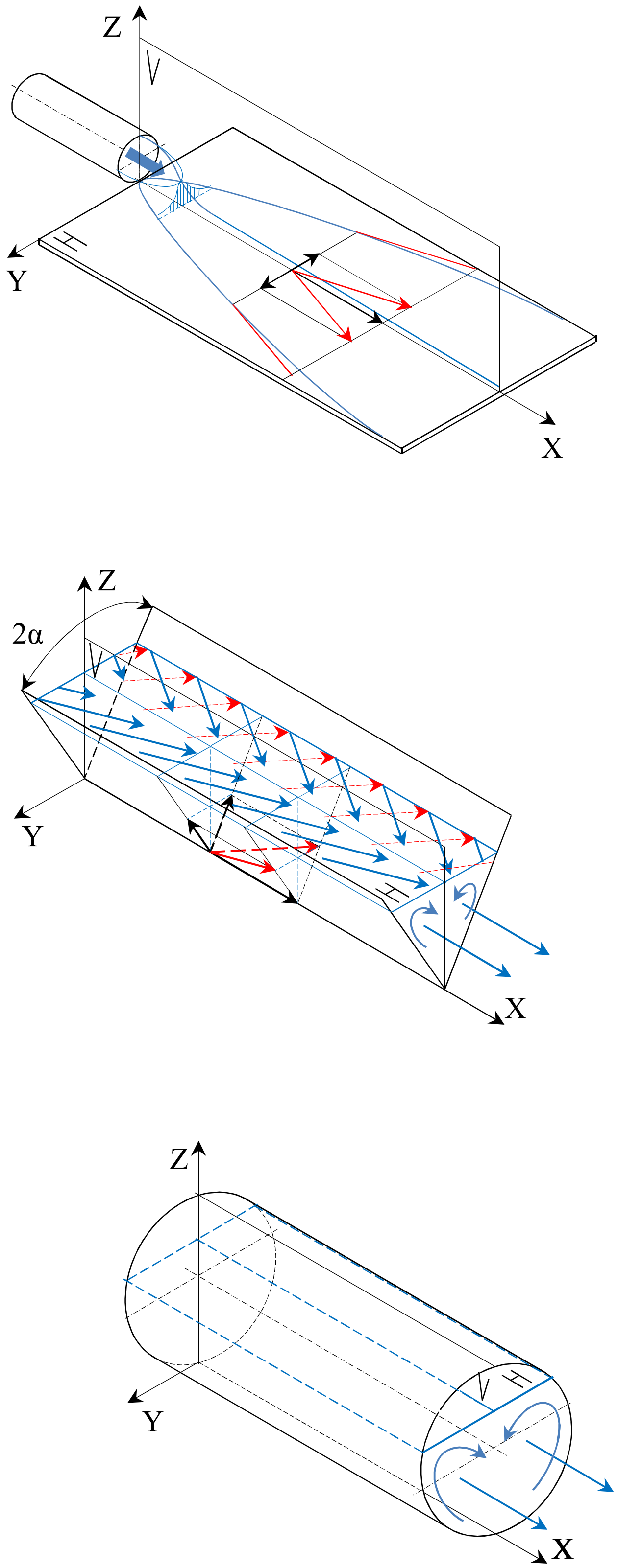


Fig.7

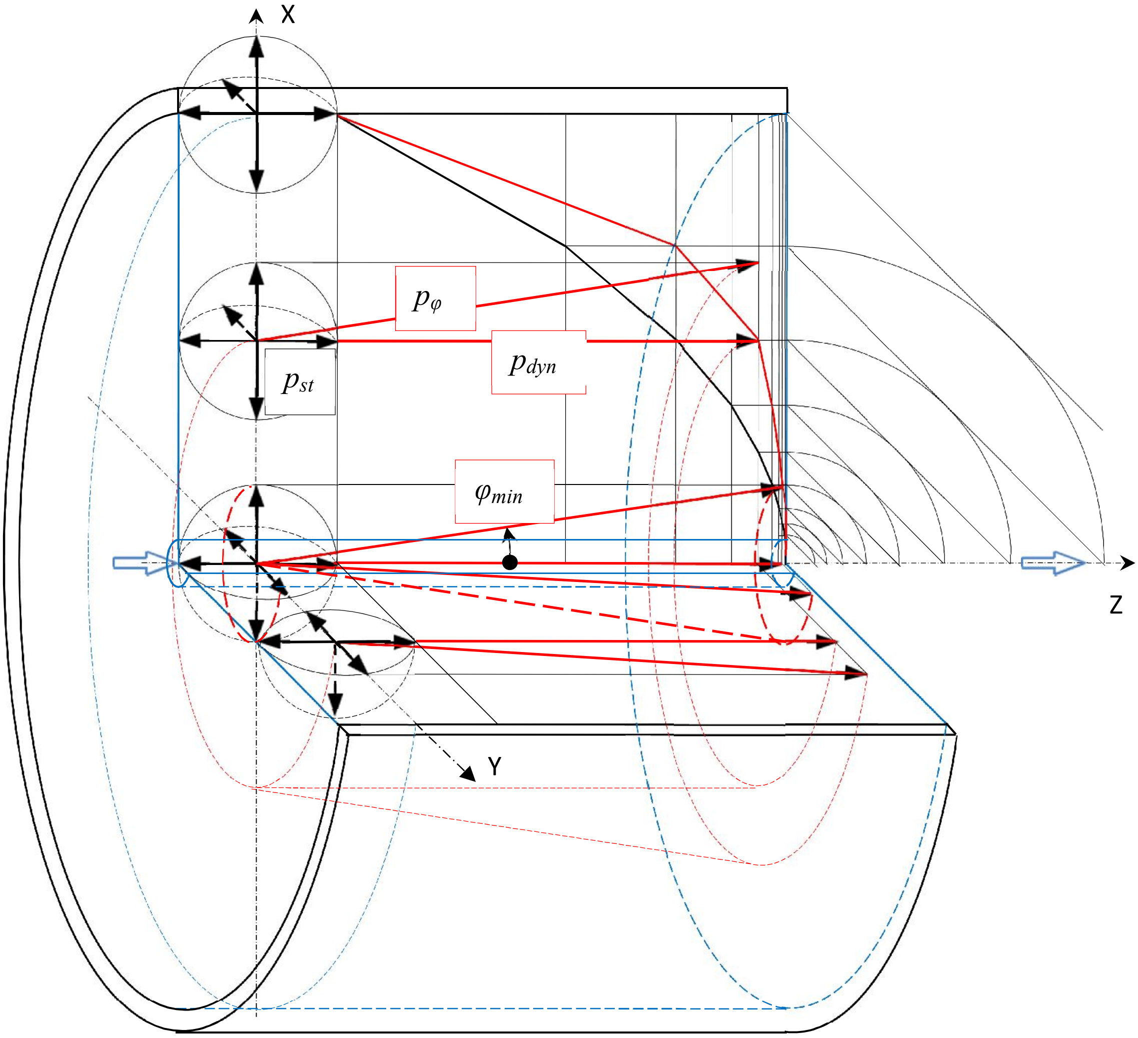
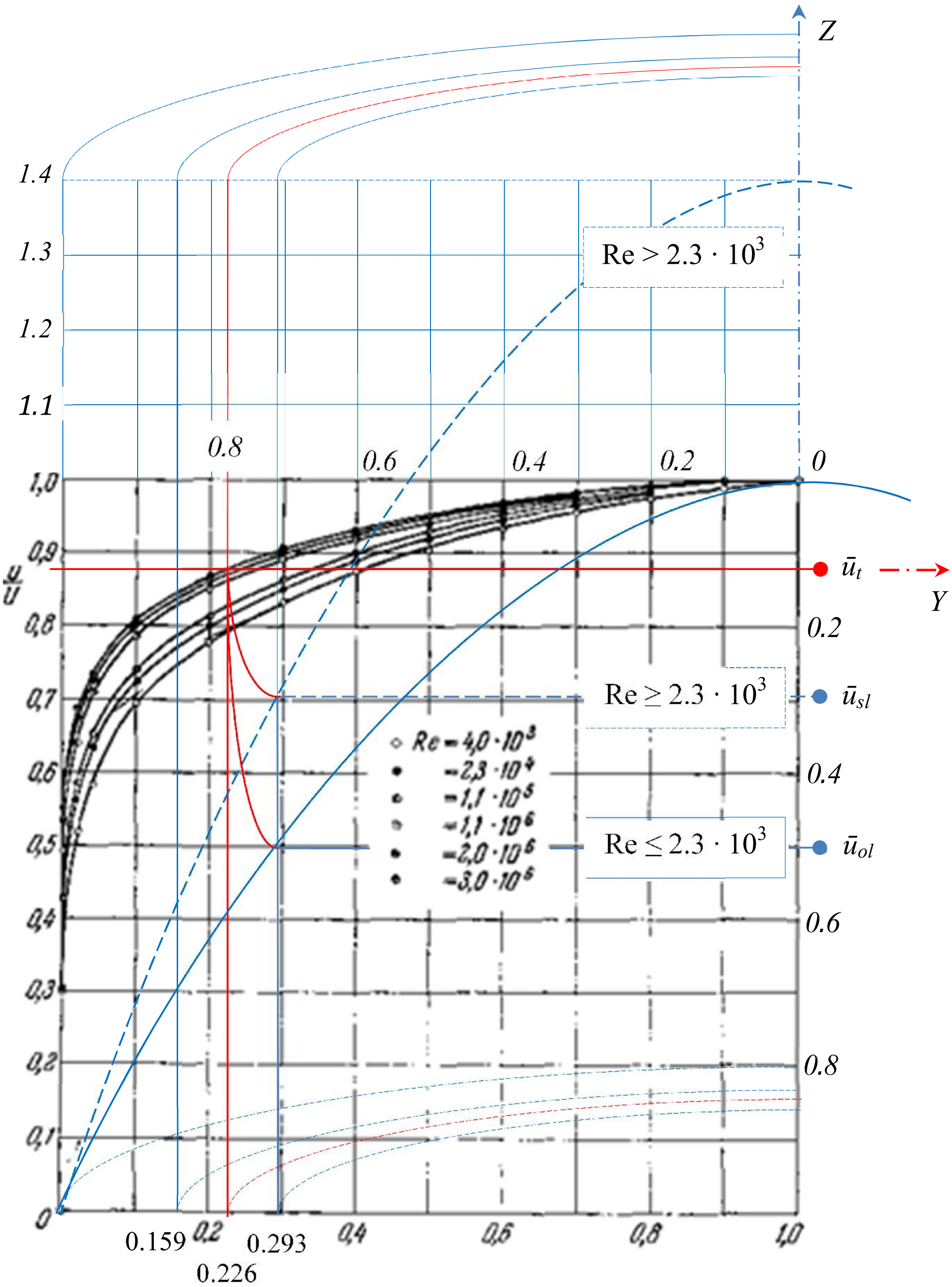


Fig.8:

R $R \geq r \geq 0$	$p_{st}(r) = const$ $p_{dyn}(r) = vario$	$p_{st} + p_{dyn} = p_{sa}$	$p_{\varphi} = [(p_{st})^2 + (p_{sa})^2]^{0.5}$
$v \sim f(r^2)$ $p_{dyn} \sim f(r^4)$	$p_{st}(l) = vario$ $p_{dyn}(l) = const$	$\varphi = \arctan p_{st} / p_{sa}$	$\varphi_{min} = \varphi_{r=0}$ $\varphi_{max} = \varphi_R = 90°$



The diagram in black lines by J. Nikuradse (1932)
is adopted from H. Schlichting's book [4, Fig.20.2]

Fig.9

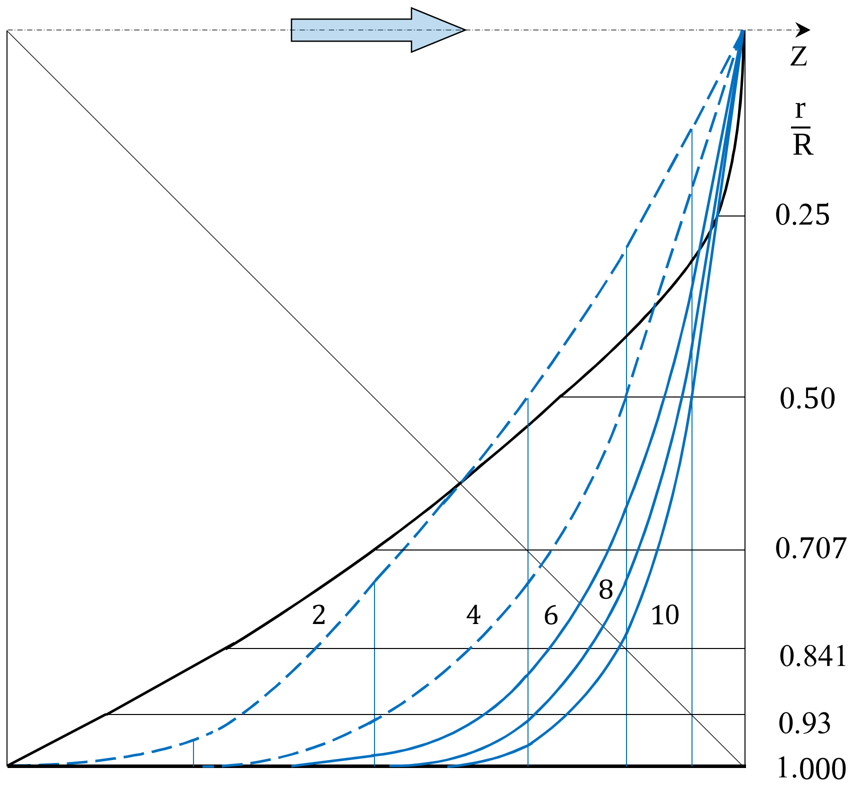


Fig.10