Modern fluid motion physics
Part 2: the Euler momentum conservation equation solution

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Solution of Euler momentum conservation equation for the real fluid - gas and liquid - stream in a pipe flow element is for the first time obtained on the basis of in essence new physically adequate approach to a problem on a contact interaction of the fluid with bodies. The flow element, system is: pipe, tube, mouthpiece, diffuser, etc. and its combination. Integration of the differential equation has led to the distribution law of static pressure along the flow element and one has proved the elementary algebraic solution, obtained in previous article by one of these authors [1]. But in contrast to previous solution, the integral solution allows to describe the real fluid motion under non-stationary conditions in combination with action of any time-varying physical factors: a pressure drop, applied to the fluid stream in the flow element, roughness of its wall, its cross-section area, and also the heat exchange with the streamlined surface, technical work, additional flow rate.

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Nomenclature

D – internal diameter of the flow element;
L – general metric length of the flow element;
$\bar{L}$ – general caliber length of the flow element, L/D;
l – metric length of stream from the outlet section of the flow element up to running point;
p_0, p_h – pressure at inlet and at outlet of the flow element respectively;
$p, p_{st}(l)$ – static pressure along the fluid stream;
V – a stream velocity, determined by weight flow;
ζ_{in} – coefficient of local hydraulic resistance for inlet into the flow element;
ζ_{ex} – coefficient of local hydraulic resistance for outlet from the flow element;
ζ_{loc} – coefficient of local hydraulic resistance within the limits of the flow element;
λ – hydraulic friction – Darcy -- coefficient;
ρ – fluid mass density.

Introduction

A problem on solution of the momentum conservation differential equation for real – gas and liquid - fluid stream, moving in a pipe flow element, had appeared from a moment of its publication by Euler in 1755. The problem is bound with a question: what do its solution means?

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And this new problem, in turn, demands clearing up of the physical meaning, contained in the given equation. If the equation describes notion of ideal – non-viscous – fluid stream, as Euler and his following considered, then it is arising next question: what is physical reason of a change of the correlation between two components of the equation in its left-hand side under condition of conservation of equality of its total to zero value? Apparently, the medium, which is not having the internal structural kinematic and dynamic linkage, cannot change parameters of its motion. The fluid stream, which does not possess the above-mentioned property, can be considered only as a whole with stationary values of motion parameters in all its parts by analogy with the Newton first law. All the more, such medium does not capable to interact by contact with environment. It is impossible neither to understand sense of Euler equation, nor to decide it productively because of physical emptiness of such medium, termed by ideal fluid.
In XX century, the attempt of substitution in the left-hand side of Euler equation of the static head losses along the pipe length, in accordance to Weissbach-Darcy formula, was undertaken after experimental determination of allocation of static head in the pipe water flow. Such way, on thought of hydrodynamic scientists of XX century, was to have ensures passage to consideration of real fluid, possessing such internal structural property as viscosity and capable to response to external actions. However, this way was found unproductive. Introduction in Euler equation of the third member signifies what the fluid stream, alongside with kinetic and potential energy, contains a certain, third, type of energy, bound with the contact interaction of the stream with a streamlined surface. In this form, Euler equation has ceased to be the equation as such both physically and mathematically. In reality, the problem of productive usage of Euler equation as one of basic equations for description of the fluid dynamics in the pipe flow element is in that
- to retain its form of record;
- to take into account mathematically correctly influence of the contact interaction on a correlation of kinetic and potential energy, which varies along the stream length, but the total content of these two types of energy is remained invariable;
- to know, that the solution should give the potential distribution, that is the static head in stream along the flow element length according to Torricelli-Galilei-Borda-Du Buat (TGBD) correlation for free fall and Bernoulli principle on correspondence of correlation of kinetic and potential energy of a fluid flow in the pipe to this correlation for free fall of a solid in the gravity field.
Such solution of the momentum conservation equation should have an as much as possible generalized form for real gas stream and its particular form for real liquid stream. At that these both solutions will appear fundamental for gas dynamics and hydrodynamics accordingly.

Euler equation solution

So, after almost 250 years, the Euler partial differential equation looks like

$$V(l)\frac{\partial V(l)}{\partial l}+\frac{1}{\rho(l)}\frac{\partial p(l)}{\partial l}=0. \qquad (1)$$

Basing on the conception on a contact interaction of the fluid flow with the streamlined surface, it is necessary to write up the equation (1) in the form

$$d[\rho(l)\cdot V^2(l)+p_{st}(l)]=0. \qquad (2)$$

where ρV^2 is kinetic energy and p_{st} is static head both distributed along the pipe length proportionally to intensity of a contact interaction of the stream with the pipe wall.
Out of the equation (2) it follows

$$\rho(l)\cdot V^2(l)+p_{st}(l)=const. \qquad (3)$$

The value of the constant is determined by the requirement $V = 0$ when $p_{st} = p_0$, with allowance for which the equation (3) takes particular form

$$\rho(l) \cdot V^2(l) + p_{st}(l) = p_0. \tag{4}$$

The intensity of a contact interaction of the stream with the pipe wall is expressed by dependence

$$dp(l) = \lambda \frac{1}{D} \frac{\rho(l) \cdot V^2(l)}{2} dl, \tag{5}$$

which corresponds to differential form of writing of Weissbach – Darcy formula for the fluid stream in pipe. Integration of the equation (5) gives distribution of static head along the pipe length according to intensity of a contact interaction of the stream with the pipe wall in the form

$$p_{st}(l) = \lambda \frac{1}{D} \int_0^l \frac{\rho(l) \cdot V^2(l)}{2} + C, \tag{6}$$

where the actual longitudinal coordinate, l, needs be counted out from the outlet section of the pipe. The integration constant, C, in the equation (6) is determined out of requirement $l = 0$ when $p = p_0$. In the result, $C = p_h$ and the equation (6) acquires the form

$$p_{st}(l) = \frac{\lambda}{D} \int_0^l \frac{\rho(l) \cdot V^2(l)}{2} + p_h\ . \tag{7}$$

In the result of substitution (7) into (4), and taking into account $\rho(l) \cdot V(l) = const$, we now have

$$\frac{\rho(l) \cdot V^2(l)}{2} = \frac{p_0 - p_h}{2 + \frac{\lambda}{D} \rho(l) \int_0^l \frac{1}{\rho(l)} dl + \sum \zeta_{loc}}. \tag{8}$$

In one's turn, having substituted (8) into (7), we receive the distribution law of the static head along the length of the pipe flow element

$$p_{st}(l) = (p_0 - p_h) \frac{\frac{\lambda}{D} \rho(l) \int_0^l \frac{1}{\rho(l)} dl}{2 + \frac{\lambda}{D} \rho(l) \int_0^l \frac{1}{\rho(l)} dl + \sum \zeta_{loc}} + p_h \equiv$$

(9)

$$\equiv (p_0 - p_h)\frac{\frac{\lambda}{D}\rho(l)\int_0^l \frac{1}{\rho(l)}dl}{1 + \frac{\lambda}{D}\rho(l)\int_0^l \frac{1}{\rho(l)}dl + \zeta_{in} + \zeta_{ex} + \sum\zeta_{loc}} + p_h.$$

The obtained expression (9) determines allocation of the static head along the length of the gas stream in the pipe flow element.
For the liquid stream in the pipe flow element, it is valid $\rho(l) \cdot V^2(l)/2 = const$ and the expression (8) takes the kind

$$\frac{\rho V^2}{2} = \frac{(p_0 - p_h)}{2 + \lambda\frac{L}{D} + \sum\zeta_{loc}}. \tag{10}$$

Substituting (10) into (7), we obtain the particular solution for allocation of the static head along the length of the liquid stream in the pipe flow element

$$p_{st}(l) = (p_0 - p_h)\frac{\lambda\frac{l}{D}}{2 + \lambda\bar{L} + \sum\zeta_{loc}} + p_h \equiv$$

(11)

$$\equiv (p_0 - p_h)\frac{\lambda\bar{l}}{1 + \lambda\bar{L} + \zeta_{in} + \zeta_{ex} + \sum\zeta_{loc}} + p_h.$$

Concluding remarks

Thus, later almost 250 years, physically adequate and mathematically correct solution of the momentum conservation Euler equation is obtained as one of fundamental correlations in the field of gas dynamics and hydrodynamics. The solution in its differential form quite corresponds to elementary algebraic solution [1] and in that way corroborates uniqueness of the solution. At the same time the differential approach has allowed to receive the solution suitable for determining of the state and motion parameters of compressible and incompressible fluids in non-stationary conditions under action of any time-varying physical factors: a roughness of a streamlined surface, the area of the flow element cross-section, the heat exchange with the streamlined surface, the technical work and additional weight flow of a fluid.

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[1] S.L. Arsenjev, Modern fluid motion physics. Part 1: on static pressure law in the pipe flow element, 2003