Mechanics of solid body, hydromechanics and gas dynamics in conic sections as a way for solution of the problems
Part 3: The conic sections in engineering and the Nature's phenomena

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It is adduced series of examples, illustrated the form building properties of a circumference evolvent in some engineering fields, including the flow elements and systems with real compressible fluid and incompressible air-water foam-aerosol mixture, a solid stressed state and also in some natural phenomena, including meteorology, biology, … elements of architecture, wind instruments
PACS: 47.40.+ x; 47.60.+ i; 47.85.- g; 82.70. Rr; 92.60.- e

Introduction

Successive solution of problems on elastic and plastic deformation of axisymmetric solid body up to its fracture and then of problems on hydraulics applied to a tube system with sharp exchange of its diameter opens a possibility for solution of problems in engineering fields and some natural phenomena, bound with movement of compressible fluid in a flow element, system, incompressible air-water mixture in meteorology and other, including biology, wind instrument, architecture. A leading role in solution of the problems is granted to an evolvent of circumference.

Solutions

Fig.1 shows again graphical formula of geometric compatibility of conic curves which one contains in its upper part two symmetric relative axis Z evolvents as longitudinal profile converging nozzle with minimum hydraulic losses.

Fig.2 offers two pairs of evolvents forming a longitudinal profile of converging-diverging flow element as supersonic nozzle contour.

Fig.3 offers, in its upper diagram, the air stream structure in tube immediately before transition of weight flow rate to linear dependence onto a pressure drop; in this moment, the air stream is self-focused, in its contracted cross-section has been reached sonic velocity, in tube outlet has been reached supersonic velocity, but free jet, outflowing out of tube, has not energy for its expansion and acceleration; such free jet is decelerated by system of shock waves in surrounding atmosphere. Thus the air stream motion is quite hydraulic up to M = 0.3; further, up to M = 1, the motion becomes isentropic and transonic, and then one becomes isentropic and supersonic. The stream cross-section radius at M = 1 is determined by means of classic isentropic expression

$$p_{cr} = p_0 \left(\frac{2}{k+1}\right)^{k/(k-1)} = A_{cr}/A_t,$$

where A_{cr} and A_t is a cross-section area of the stream at M = 1 and tube correspondingly.
Let be $A_t = \pi \cdot R_t^2 = 1$, i.e. $R_t' = \sqrt{1/\pi} = 0.5642$. And, correspondingly, let be

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$$A_{cr} = \pi \cdot R_{cr}^2 = 0.5283, \text{ i.e. } R'_{cr} = \sqrt{0.5283/\pi} = 0.41,$$

and then $$R_{cr} = R_t\, 0.41/0.5642 = 0.727R_t.$$

The obtained result corresponds to radius, where mean velocity acts in laminar flow, with error no more +3 per cent. Longitudinal profile in the stream contraction zone is described by two evolvents and by part of the right one-hollowed hyperboloid of rotation between its.
Lower diagram offers the developed supersonic motion of air stream out of tube: now further increase in pressure drop leads to direct proportional increase in weight flow rate of air stream, as it is offered in one of previous articles of the author and his collaborator Y.P. Sirik [1]. Now

$$A_{cr} = \pi \cdot R_{cr}^2 = 1.893, \text{ i.e. } R'_{cr} = \sqrt{1.893/\pi} = 0.7762,$$

and then $$R_{cr} = R_t\, 0.7762/0.5642 = 1.376R_t.$$

Thus the air stream radius in its critical section exceeds the tube radius in $\sqrt{2}$ times with error no more -3 per cent. Such free supersonic jet, outflowing out of tube, has reserve of potential energy for its expansion and acceleration proportional to the given quantity p_a/p_0. The expansion section of free supersonic jet contains its core and superlayer, enveloping the core. A profile of both these parts of jet is described by arches of a circumference evolvents. As a matter of fact, the profile can be described by some curve from set of spirals; at the same time equation of a circumference evolvent in natural form has a kind $R_i = \sqrt{2r_0(L - l_i)}$, where R_i is a curvature radius of evolvent, r_0 – radius of circumference as constant element forming the evolvent, L and l_i – certain and running length of evolvent, quite corresponding to classic formula $v_i = \sqrt{2g(H - h_i)}$ as a base for hydro- and gas dynamics at least. In these both cases the question is on movement along shortest way in field of difference of potentials. It should be noted two features in the flow structure: sublayer in tube gives place to superlayer in free supersonic jet, and convex profile of a stream velocity in tube gives place to concave profile of velocity in free supersonic jet.

Fig.4 offers an acceleration section of free supersonic air jet, outflowing out of converging nozzle into surrounding normal atmosphere. The nozzle profile is formed by a circumference evolvent. A profile of the supersonic jet is geometric place – locus – of the jet full head vector. The vector is formed by two mutually perpendicular components:

radial $$p_r = \rho\, c^2/2,$$

and axial $$p_Z = \rho\frac{c^2}{k} + \rho\frac{v^2}{2} = \rho c^2\,(2 + kM^2)/2k,$$

in sum $$p_{fv} = \rho\frac{c^2}{k}\sqrt{1 + \left(\frac{2+kM^2}{2}\right)^2},$$

where c is a running sound velocity, $k = 1.4$ - adiabatic exponent for air, $M = v/c$. The running value of the vector p_{fv} inclination to the jet axis at atmospheric pressure p_h

$$\alpha_h = arctan\frac{p_r - p_h}{p_Z - p_h} = arctan\frac{1 - p_h/p_r}{[(2 + kM^2)/2] - p_h/p_r}.$$

The greatest quantity of the angle corresponds to the jet beginning cross-section at M = 0.3, and the vector will be parallel to the jet axis in the ending cross-section of the acceleration section, where $p_r = p_h$. A known in advance radius R_{min} in outlet cross-section of converging nozzle and radius R_{max} of the jet in the ending cross-section of the acceleration section, determined by means of isentropic gas dynamic functions, determine radius of initial circumference for evolvent

$$R_0 = (R_{max} - R_{min})/(\alpha_{max} - sin\alpha_{max}).$$

A surface of rotation, formed by the evolvent arch, circumscribes the acceleration section of free supersonic air jet, and at the same time the surface is a form of shell for diverging part of supersonic nozzle.

Fig.5 offers longitudinal profile for supersonic nozzle, constructed by means of three evolvents of three circumferences; three blue lines in right side of the diagram are adopted from previous fig.4 for comparison. This supersonic nozzle profile is a limit of a possible, what can be obtained by means of the modern hydro- and gas dynamics methods.
At the same time it should be understood that the gas stream in a flow system of supersonic nozzle represents a self-excited oscillatory system: we can observe simultaneously a slow flow at M < 0.15 as subsubresonance, at 0.15 < M < 0.3 as subresonance, at 0.3 < M < 1as transresonance, at M = 1 as resonance and at M > 1 as superresonance.

Fig.6 shows a profile of supersonic jet at very small ratio, p_a/p_0.

Fig.7: an initial water jet, freely outflowing out of tube, spreads radially on a disk surface; they are showed fields of trajectories of motive forces: in kind of two sets of cylindrical helices at + 45° and - 45° on the initial water jet, of two sets of logarithmic spirals + 45° and - 45° on the disk and also in kind of straight lines at + 45° and - 45°, forming surface of the right one-hollowed hyperboloid of rotation, and helical spirals on surface of circular evolventoid of rotation – in passage from initial jet to its radial flow on the disk. Just such geometric structure of trajectories of motive forces ensures physical adequacy, mathematical correctness and also one is the only possible way of reproduction of a motion form in every considered concrete case – in full accordance with I. Newton's advice, given 333 years ago.

Fig.8: the same fields of trajectories of motive forces and spreading initial jet on surface of disk we observe in the case of a fountain jet.
Fig.9 shows the same example as in fig.7 at velocity of the water jet decreased up to a forming of hydraulic jump. The same fields of trajectories of motive forces and spreading initial jet on surface of the disk accompany this flow.

Fig.10: the same fields of trajectories of motive forces and spreading initial jet on surface of disk we observe in the case of a fountain jet.

Fig.11 shows field of trajectories of motive forces in kind of two sets of logarithmic spirals at + 45° and - 45° at invisible radial motion of a detonation front in acetylene–oxygen mixture from thin tube on plano-cylindrical bottom of chamber closed by glass with photographic plate; the photo is adopted from R.I. Soloukhin (1963) monograph [2].

Fig.12 shows field of trajectories of stresses in kind of two sets of logarithmic spirals at + 45° and - 45° in heavy-walled tube under high internal pressure; the photo had been obtained by L. Hartmann (1896) and one adopted from A. Nadai [3]; and field of deformation in the tube is formed by its invisible here radial displacement.

Fig.13 offers one of the simplest constructions of a foam-aerosol tubular scrubber with its caliber height L = 15D; the construction contains in its lower part a bladed whirler and diaphragm as it is showed in the right; a water feeder is placed at upper end of the tube; interaction of ascending whirled stream of combustion products with descending thin jets of water forms by means of the diaphragm a column of foam-aerosol structure with its height *l* = 11D. Low flow rate of fresh water is stipulated by high wet of combustion products. Mean velocity of gas stream is 7 m/s. The construction turned out unserviceable for coal-powdered fuel, and at the same time a process of forming of foam-aerosol structure turned out very similar to such natural phenomenon as origin of tornado.

Fig.14 offers diagram of consecutive process: from initial appearance of ascending stream of mild and wet air out of local heated sea surface with it twirling under Coriolis' force, what is accompanied by forming of twirling stranded jets, to a forming of steam condensate on height of clouds, then to a forming of foam-aerosol structure mixed with snow on height about 5 – 7 kilometers, then drift-glide with keeping of gyration under high-altitude wind and at last reducing altitude with additional twirling by Coriolis' force and with forming of storm zone: hail, cold rain and wind near land or sea surface; when the reducing altitude is stopped at height > 800 meters, then probability of tornado strongly increases. In upper left part of the diagram is showed a view from above onto upper part of whirlwind, constructed by means of circumference evolvent; two red concentric circumferences is view from above onto right one-hollowed hyperboloid of rotation, where straight line, as generatrix of the hyperboloid, is smoothly connected with evolvent. A body of tornado is formed by stranded jets, which ones combine its own gyration with gyration round the tornado axis: the first gyration promotes suck off air from end of tornado body and the second gyration creates centrifugal forces determining diameter of tornado body itself. In terms of electromagnetism, such spontaneously appearing and sufficiently stable combination of a fluid motion in tornado is called by solenoidal structure of the flow.

Fig.15 offers diagram of cyclonic formations relative to equatorial belt and near-equatorial wind. In lower part of the diagram is showed a photo of cyclonic formation above Indian Ocean made by artificial satellite "Cosmos-144" 11th April 1967; the photo is adopted from M.S. Fomitchev book [4].

Fig.16 offers profiles of two eggs: hen (black and red lines) and duck (blue lines); the first contour is constructed by means of two sections of evolvent of one circumference with focuses $F_1$and F_2 and the second contour is constructed by means of two the same "Egyptian" (3; 4; 5) triangle $\mathrm{OC_1C_2}$ and $\mathrm{OC_1'C_2'}$.

Fig.17 offers the same two profiles of eggs, what is offered in previous fig.16; now these is constructed by means of arches of circumferences; in lower part of the diagram is showed cross-section of aqueduct driven through rock: we see again "Egyptian" triangle.

Fig.18 is devoted to well-known Indian memorial Taj Mahal (1630 – 1652): the harmonic building is crowned by cupola-bud of flower (but not onion) in which petalous part in kind of evolventoid of rotation envelops its core part in kind of brilliant. Arch is made by a pair of parabolas of second power – in the limits y = x^2 = 1 – as equivalence of the circumference evolvent applied for cupola; here is placed second brilliant equigraphic to the first. Central tower with evolventic cupola is a symbol of masculine gender and the entrance in mausoleum under parabolic arch is a symbol of feminine gender. In the result of it, humanistic unity of masculine and feminine genders, presented in painting by Raffaele Santi in his Sistine Madonna, after almost 110 years it had found its expression in architecture of Indian mausoleum.

Fig.19, in its upper part, offers longitudinal profile of outlet part of trumpet in kind of evolventoid of rotation; and its lower part shows, in consecutive order, the evolvents of a straight line segment with its length, L, an equilateral triangle and square with its side also equal L – these spirals are constructed by means of arches of circumference; in contrast to them, evolvent of circumference is spiral with continuous change of its curvature and one completes a number of spiral lines of right polygons.

Acknowledgements
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[1] S.L. Arsenjev, Y.P. Sirik, "The Gasdynamics First Problem Solution. The Gas Stream Parameters, Structures, Metering Characteristics for Pipe, Nozzle," http://uk.arXiv.org/abs/physics/0306160, 2003

[2] R.I. Soloukhin, Shock waves and detonation in gases, State Publishing of Physic-Mathematic literature, Moscow, 1963

[3] A. Nadai, Theory of Flow and Fracture of Solids, vol.1, McGraw-Hill Book Co. Inc., Pittsburg, PA, 1949

[4] M.S. Fomitchev, Experimental hydrodynamics of NEP, State Publishing EnergoAtomIzdat, Moscow, 1989

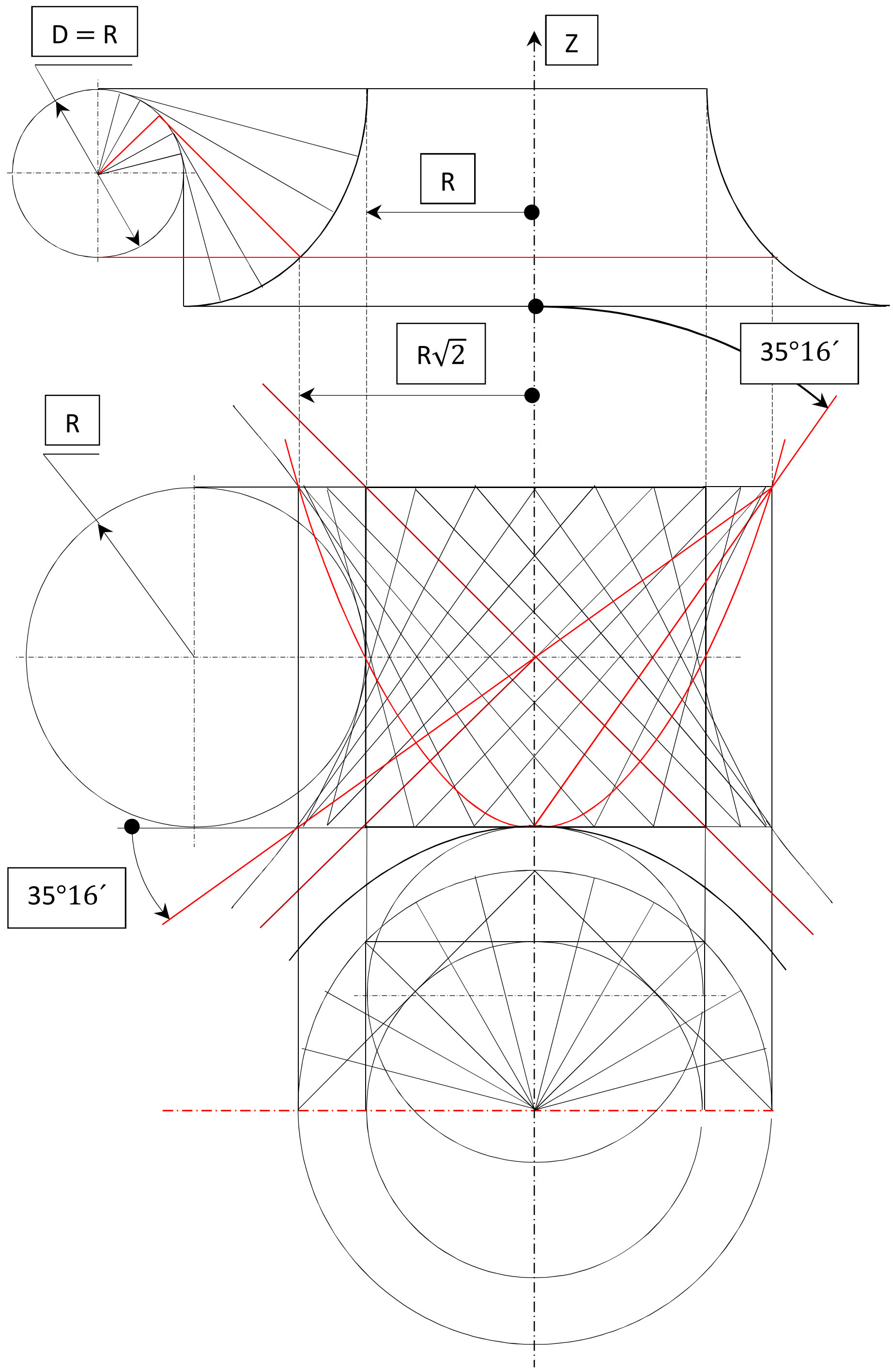


Fig.1

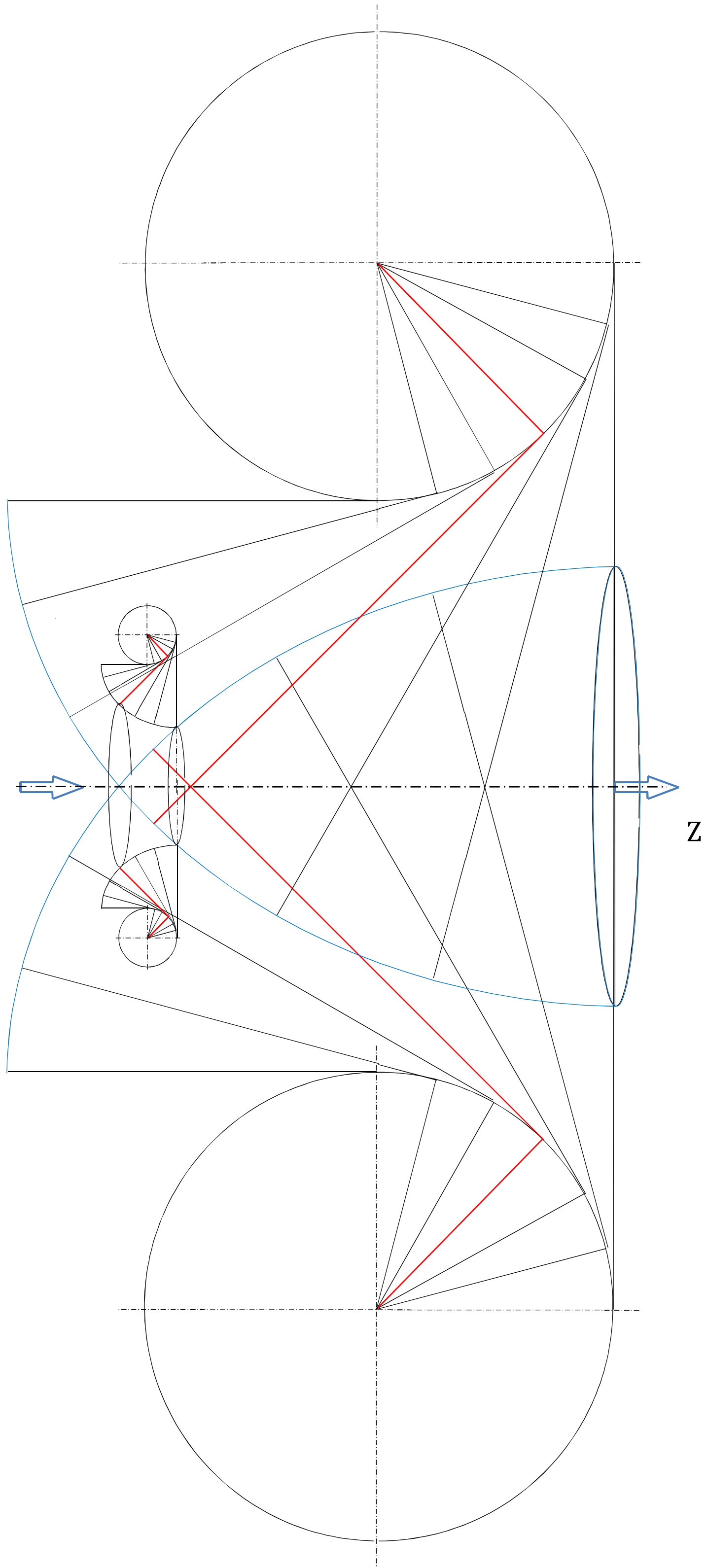


Fig.2

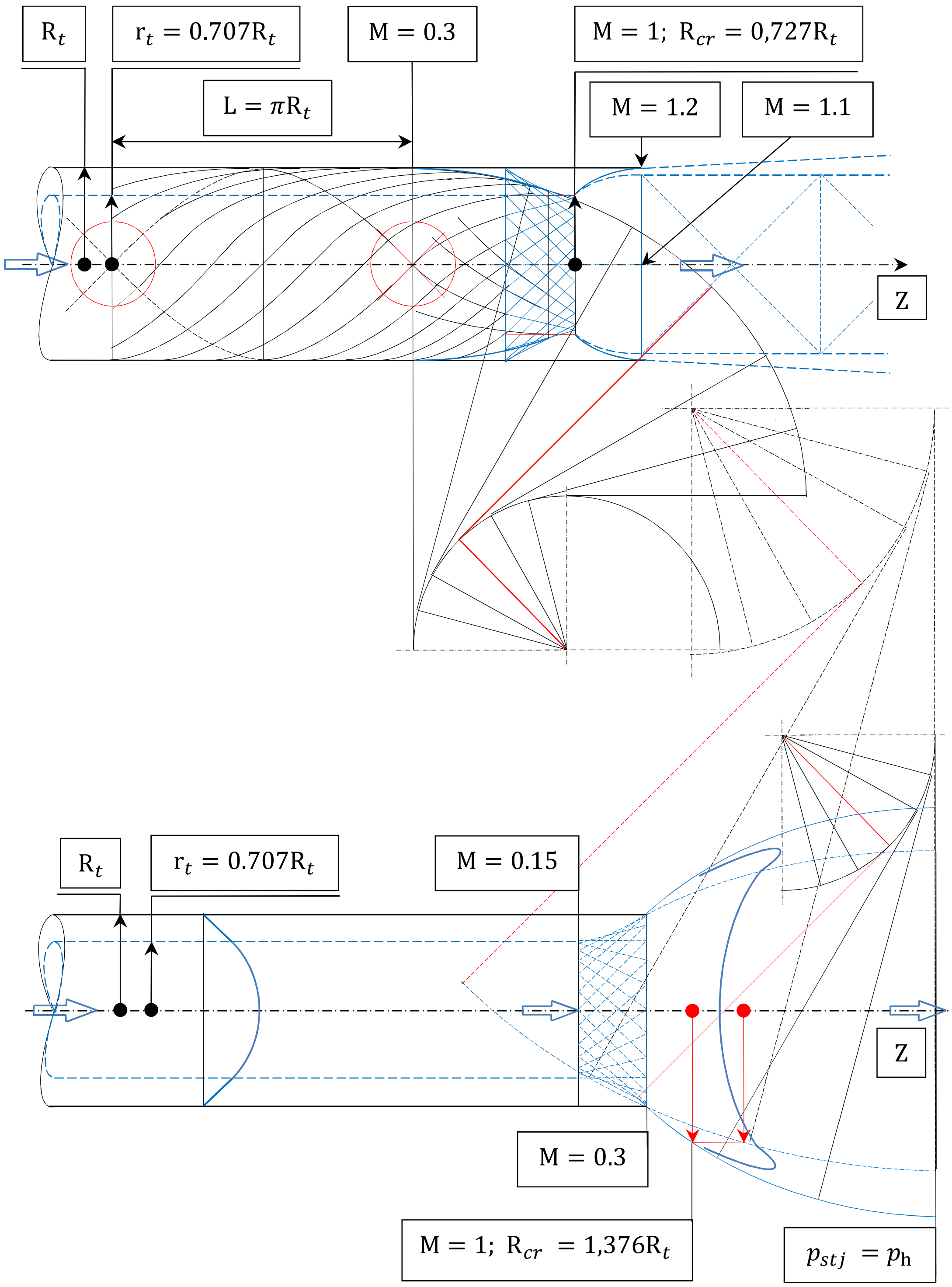

R_t
$r_t = 0.707R_t$
M = 0.3
M = 1; $R_{cr} = 0,727R_t$
$L = \pi R_t$
M = 1.2
M = 1.1
Z
R_t
$r_t = 0.707R_t$
M = 0.15
Z
M = 0.3
M = 1; $R_{cr} = 1,376R_t$
$p_{stj} = p_h$

Fig.3

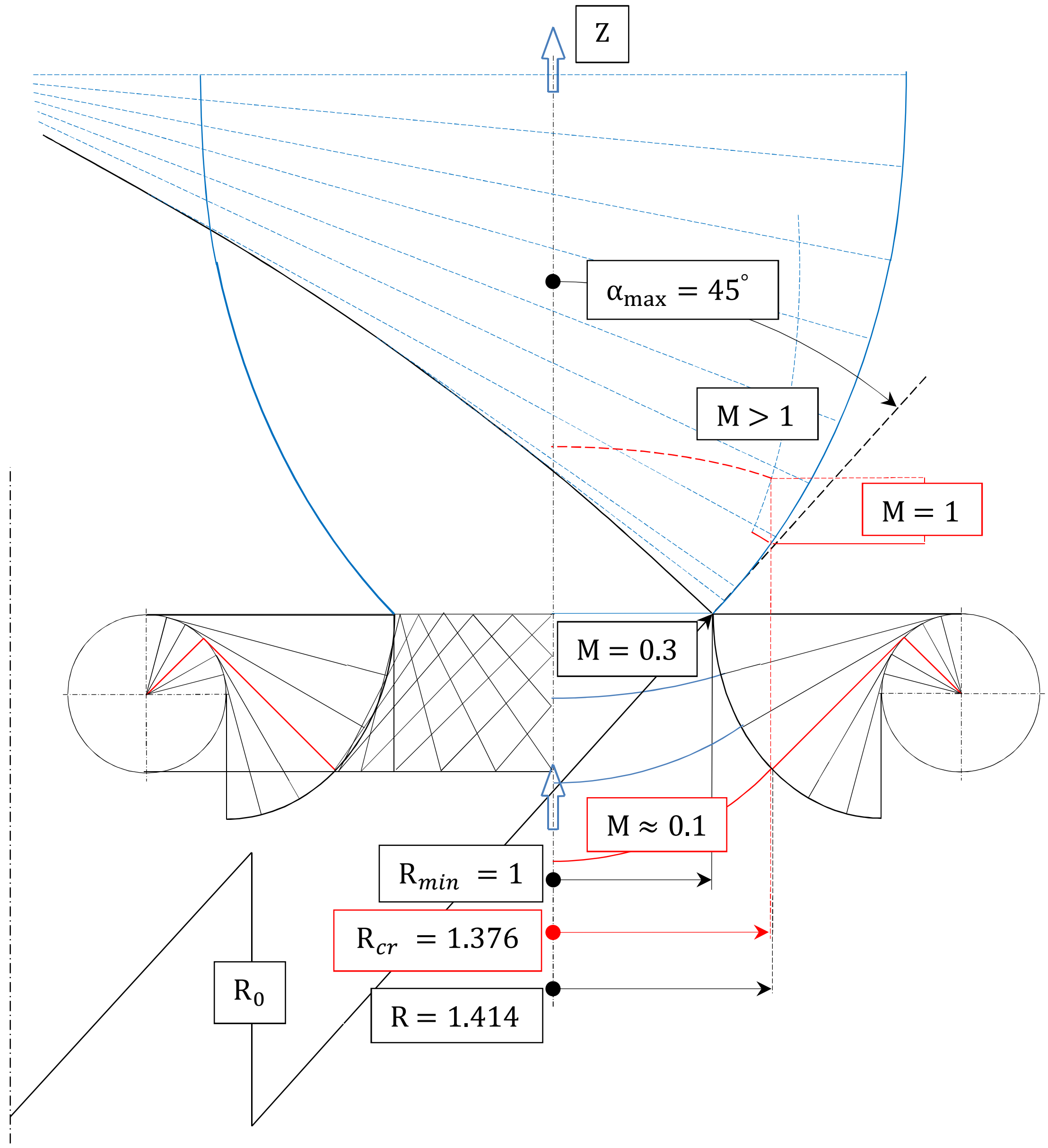


Fig.4

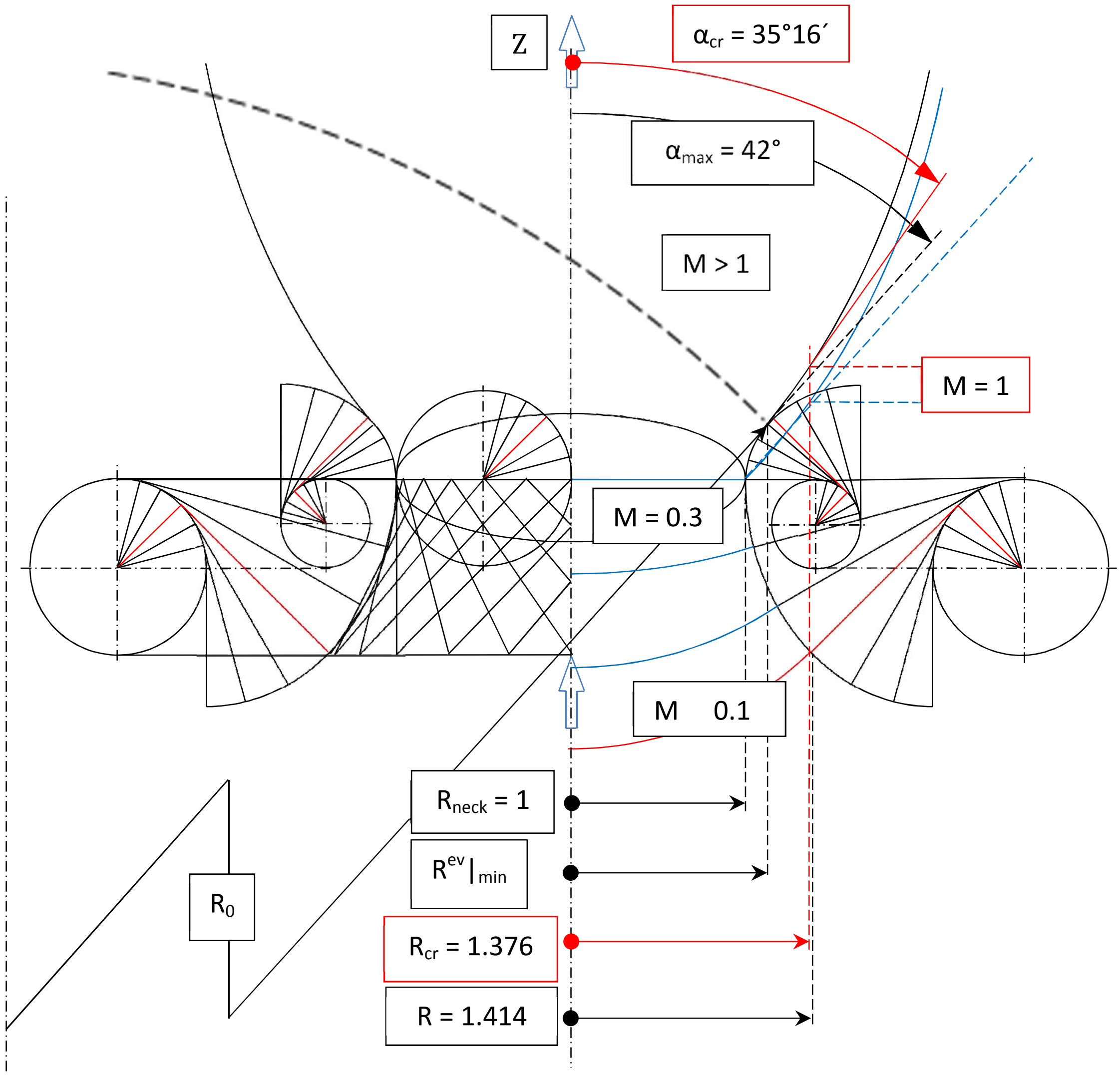

Z
αcr = 35°16´
αmax = 42°
M > 1
M = 1
M = 0.3
M 0.1
Rneck = 1
Rev|min
Rcr = 1.376
R = 1.414
R0

Fig.5

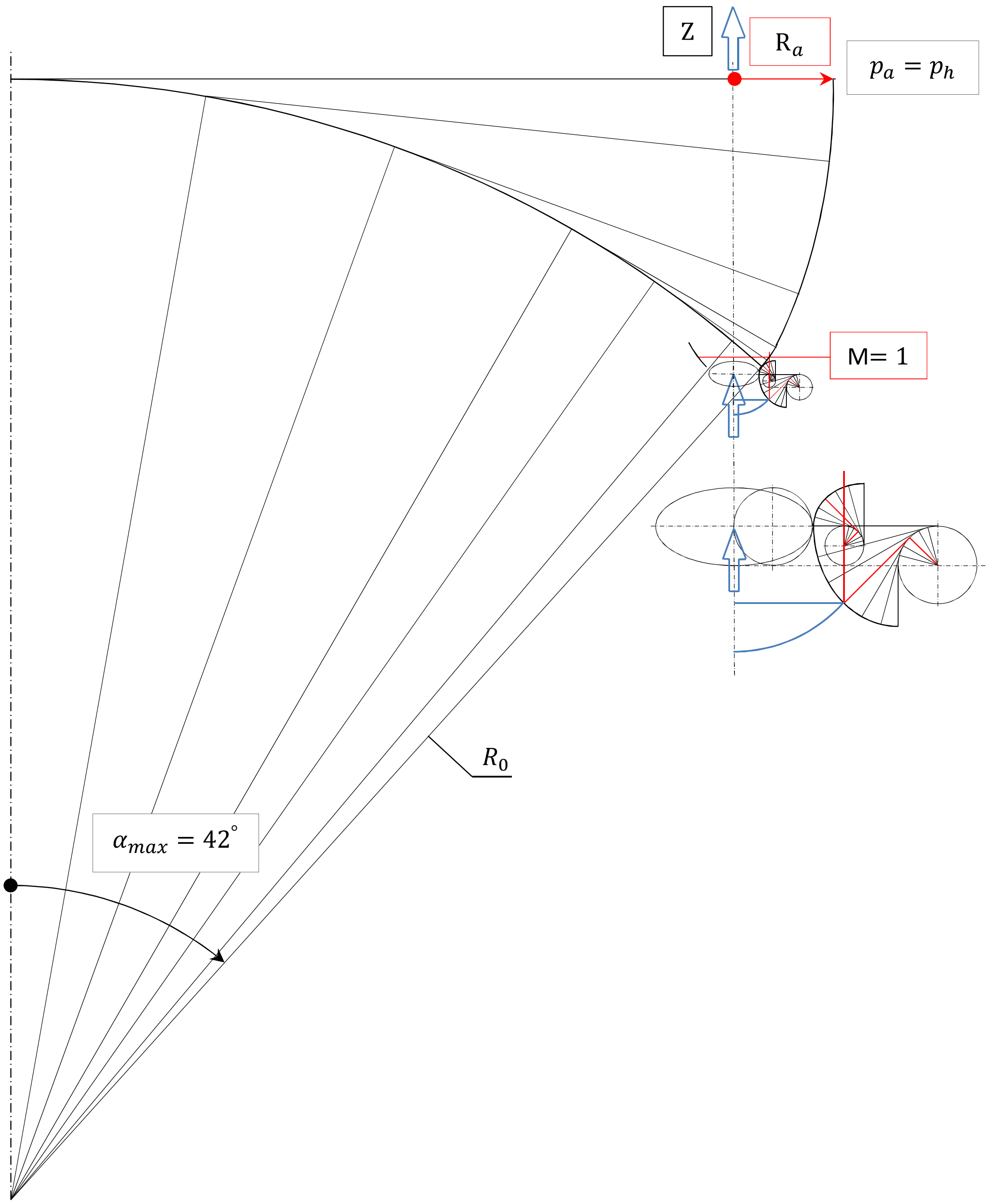


Fig.6

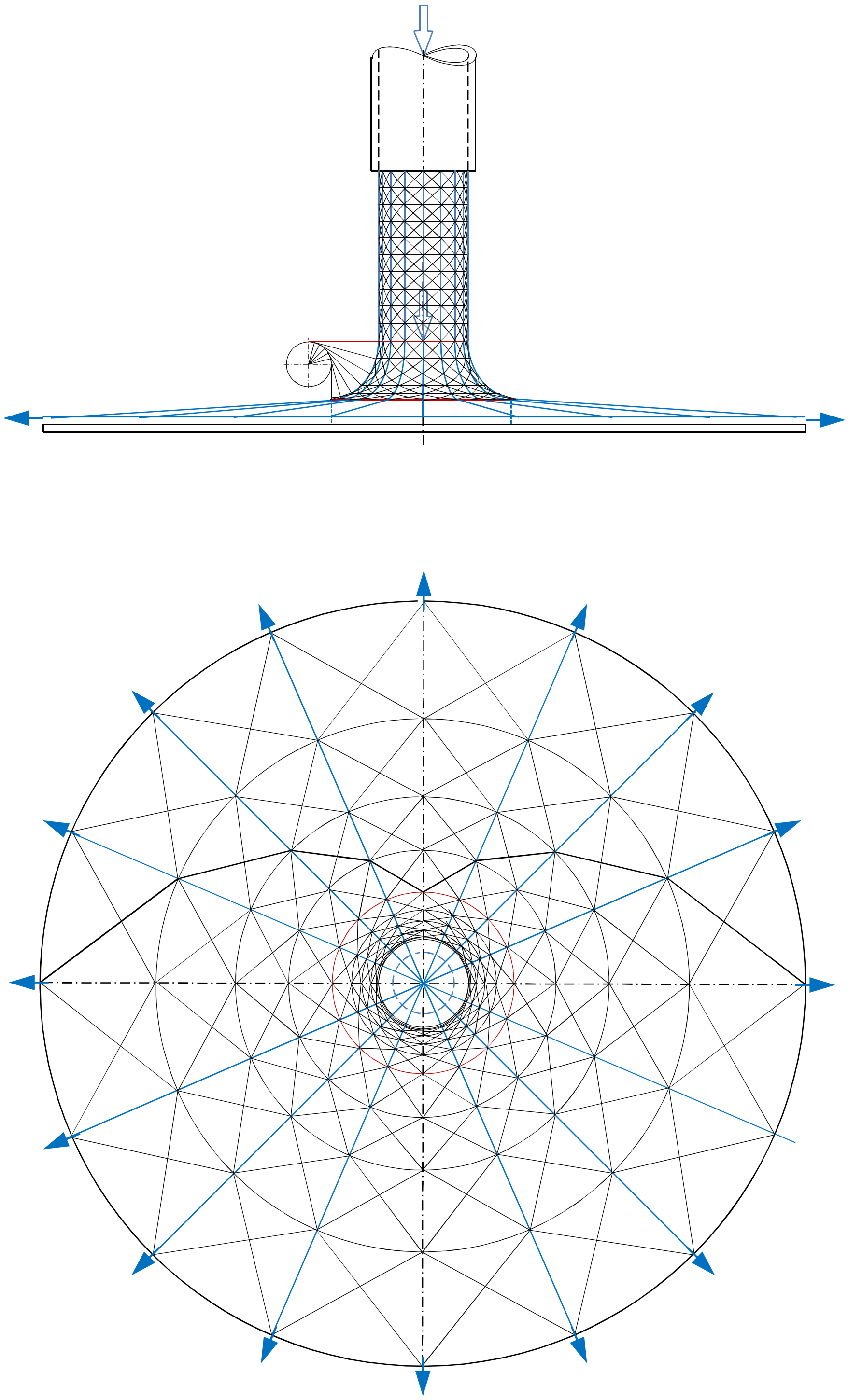
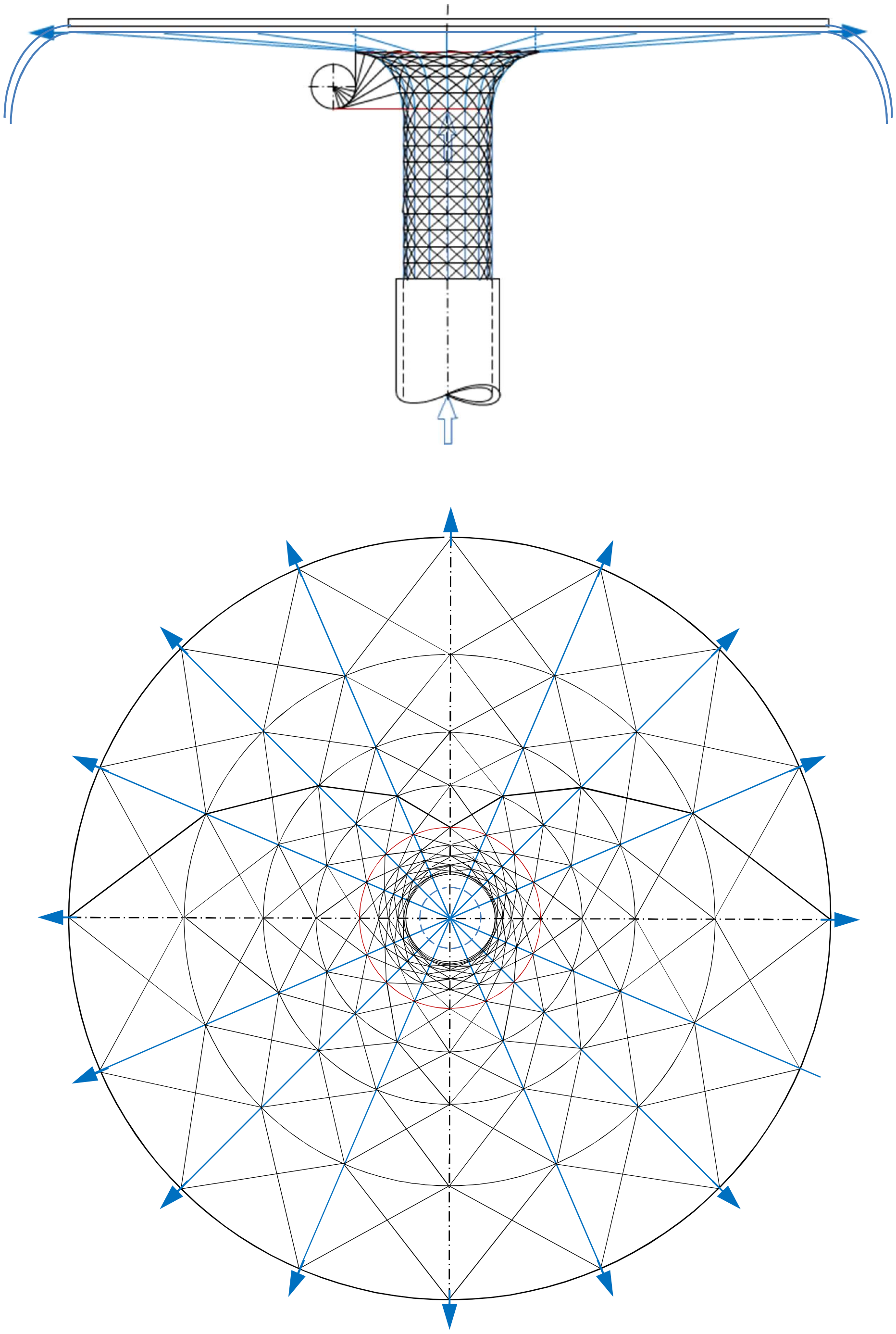


Fig.7



Fig, 8

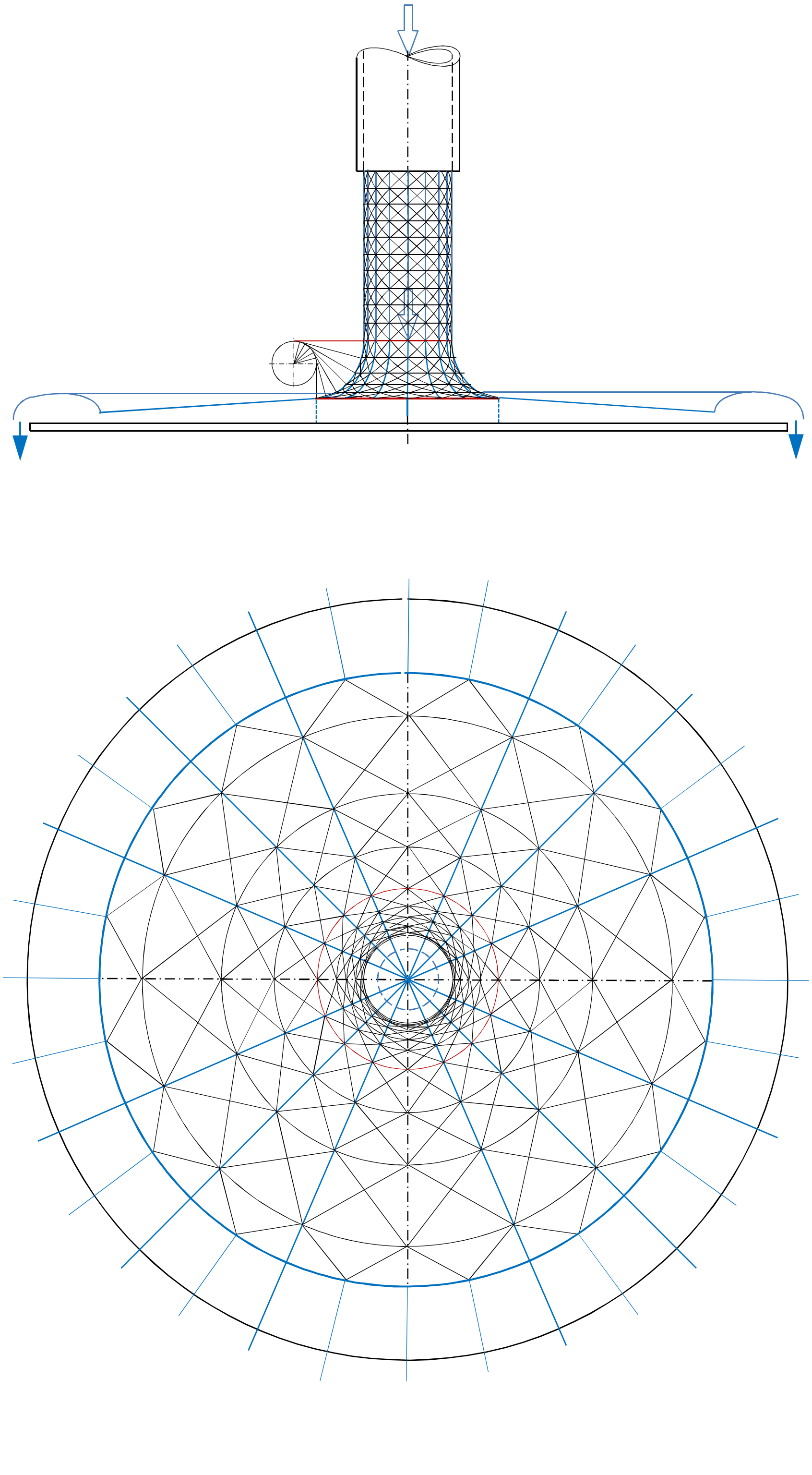


Fig.9

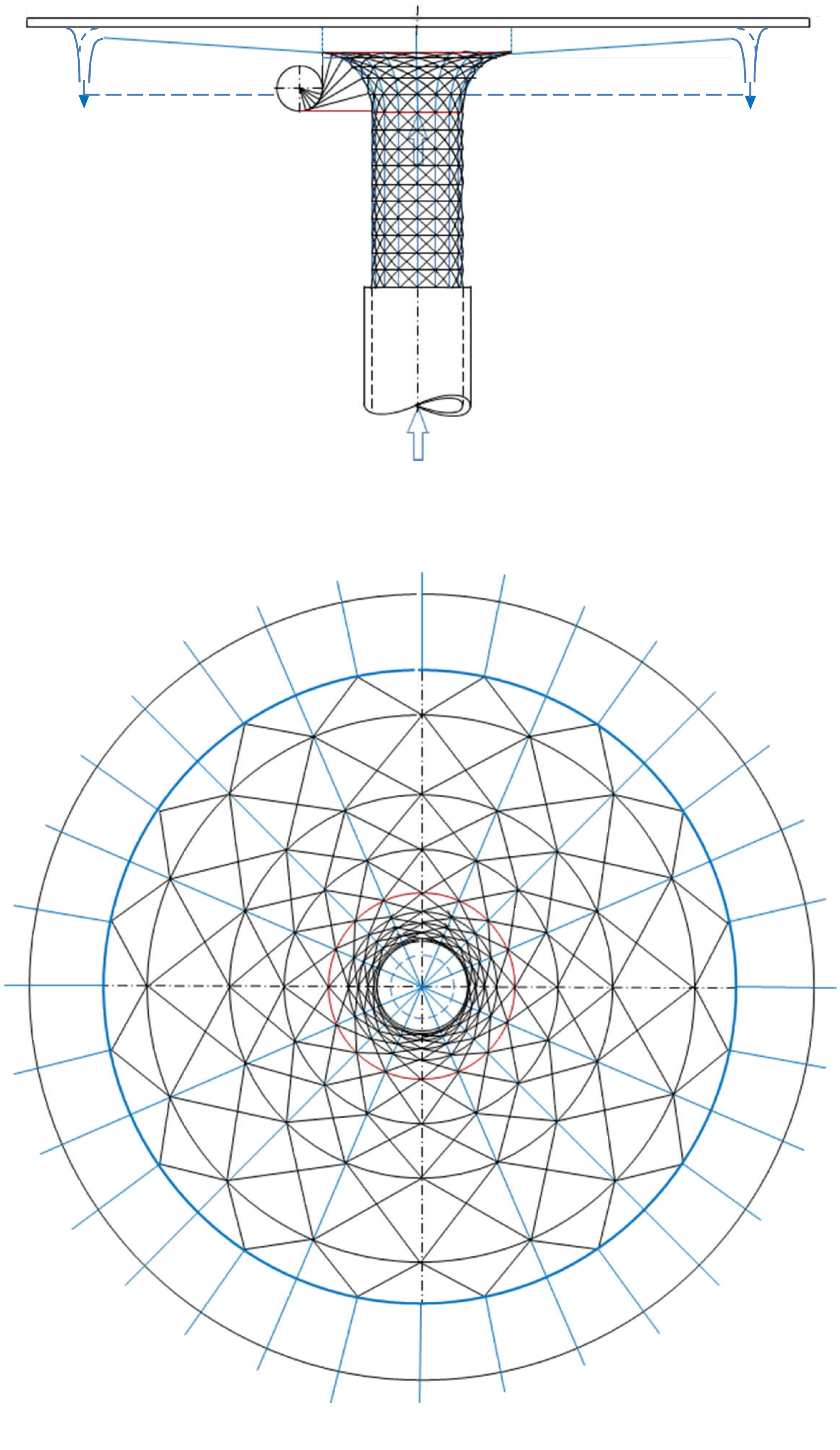
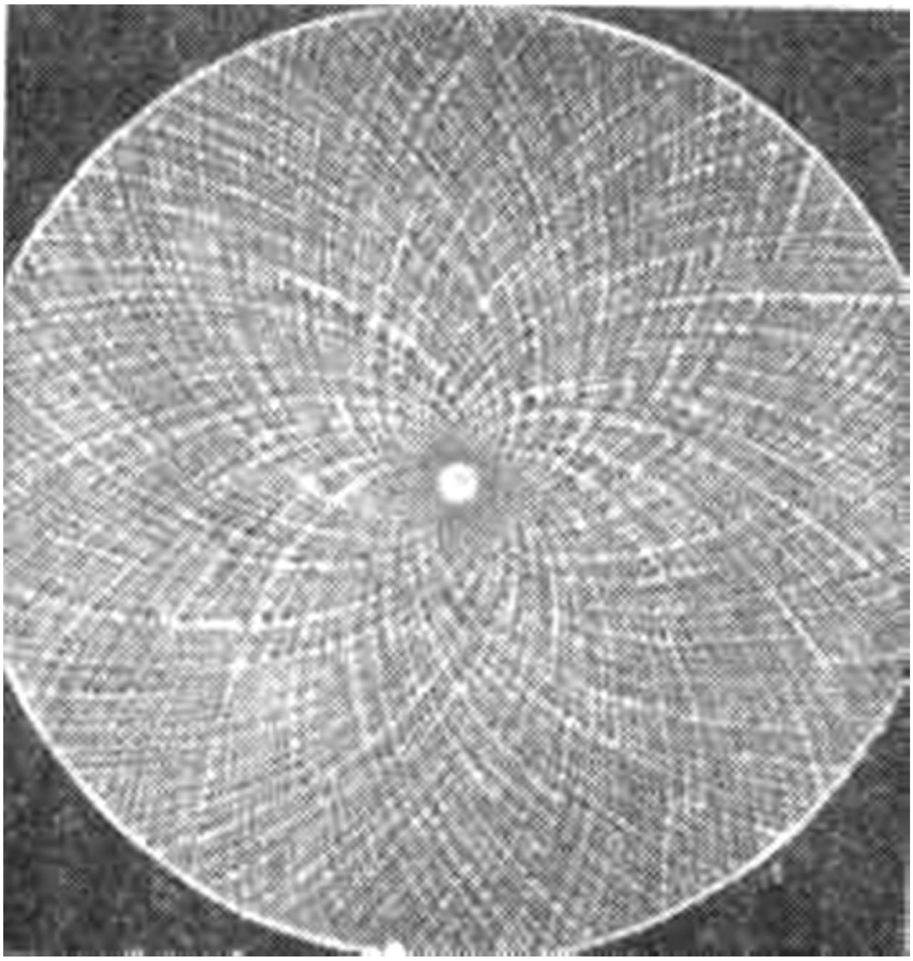


Fig.10



The photo is adopted from R.I. Soloukhin (1963) monograph [2]

Fig.11

Fig.12

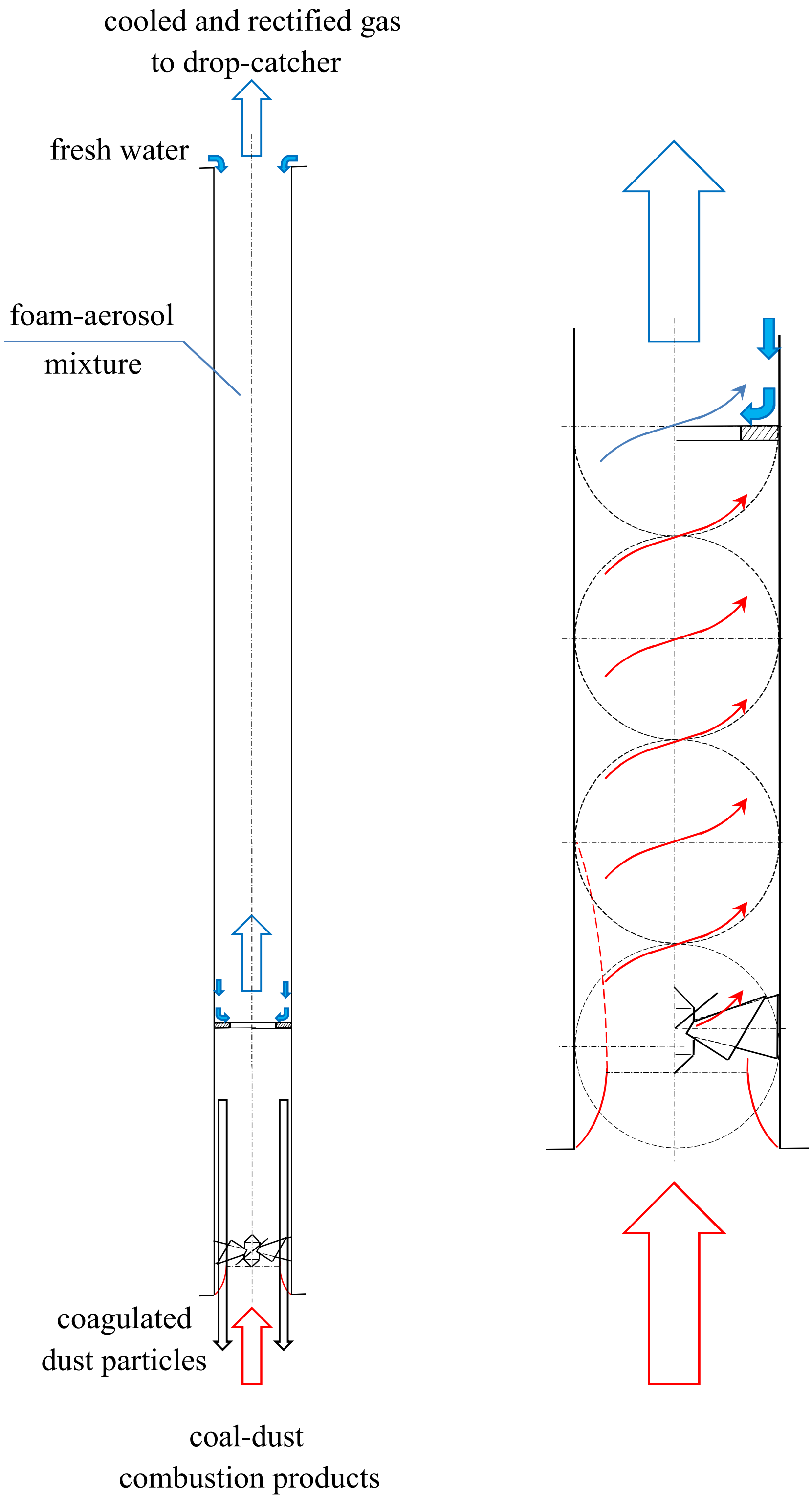

cooled and rectified gas
to drop-catcher
fresh water
foam-aerosol
mixture
coagulated
dust particles
coal-dust
combustion products

Fig.13

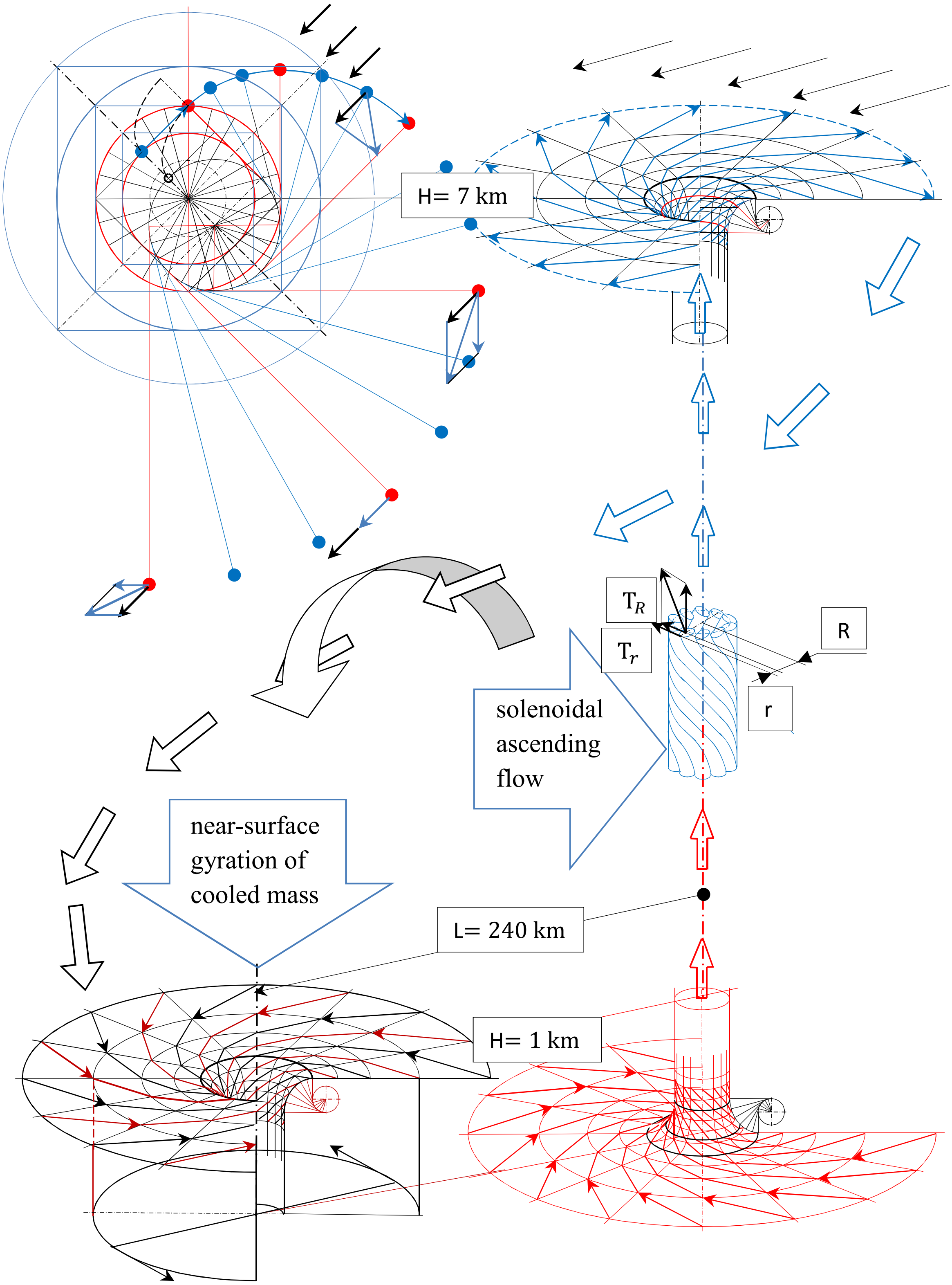

H= 7 km
T_R
T_r
R
r
solenoidal ascending flow
near-surface gyration of cooled mass
L= 240 km
H= 1 km

Fig.14

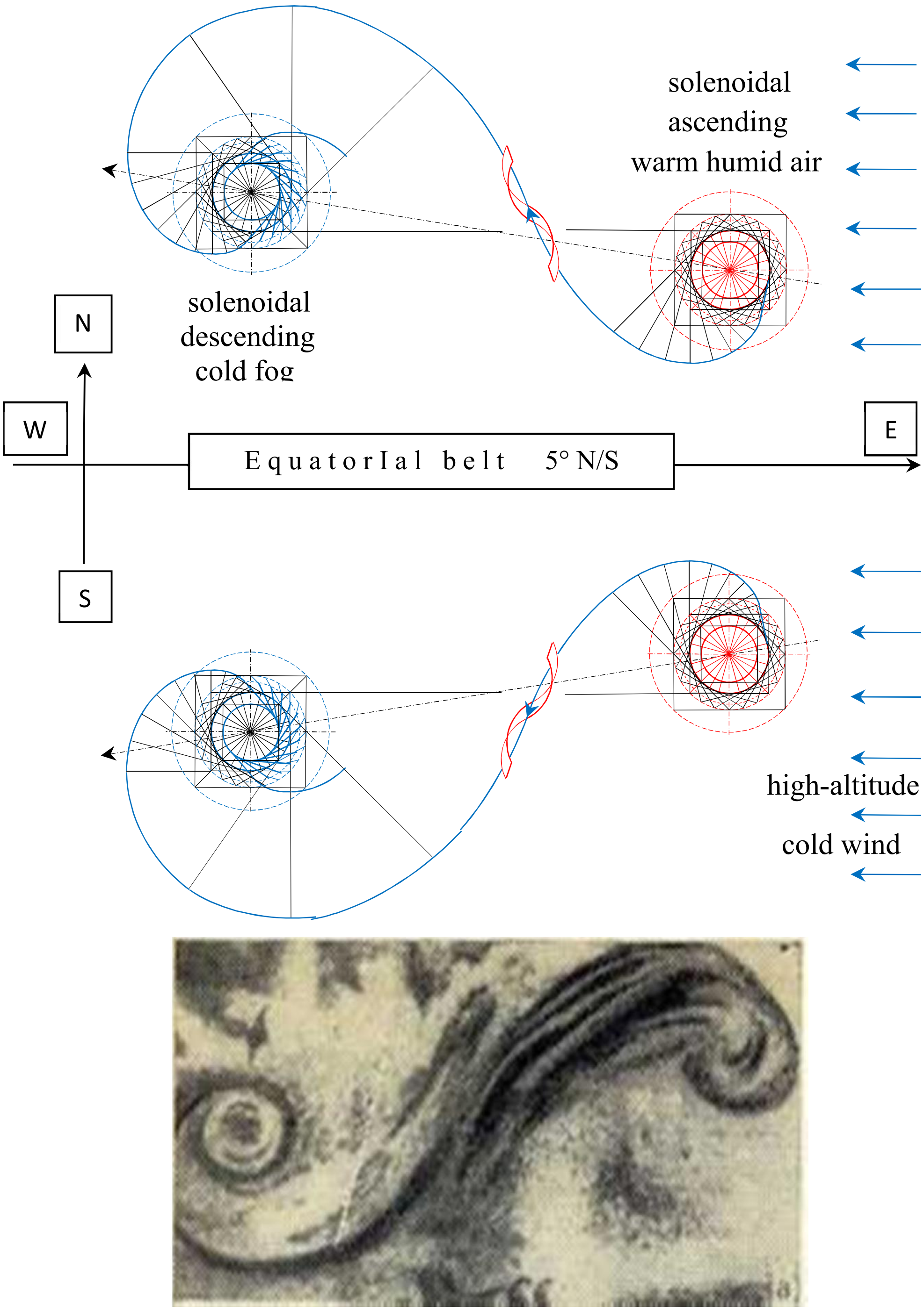

solenoidal
ascending
warm humid air
solenoidal
descending
cold fog
N
W
S
E
E q u a t o r I a l b e l t   5° N/S
high-altitude
cold wind

Fig.15

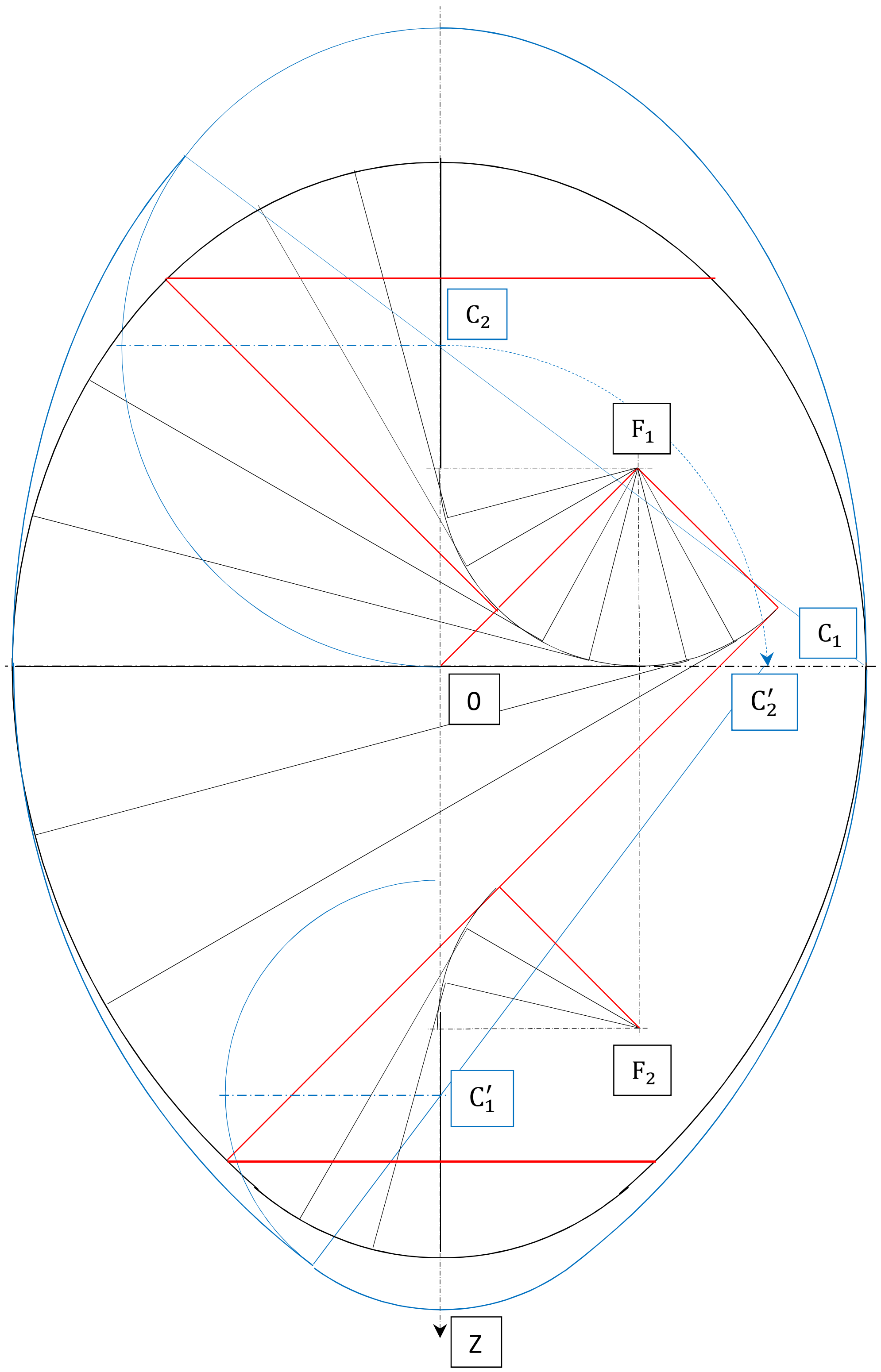


Fig.16

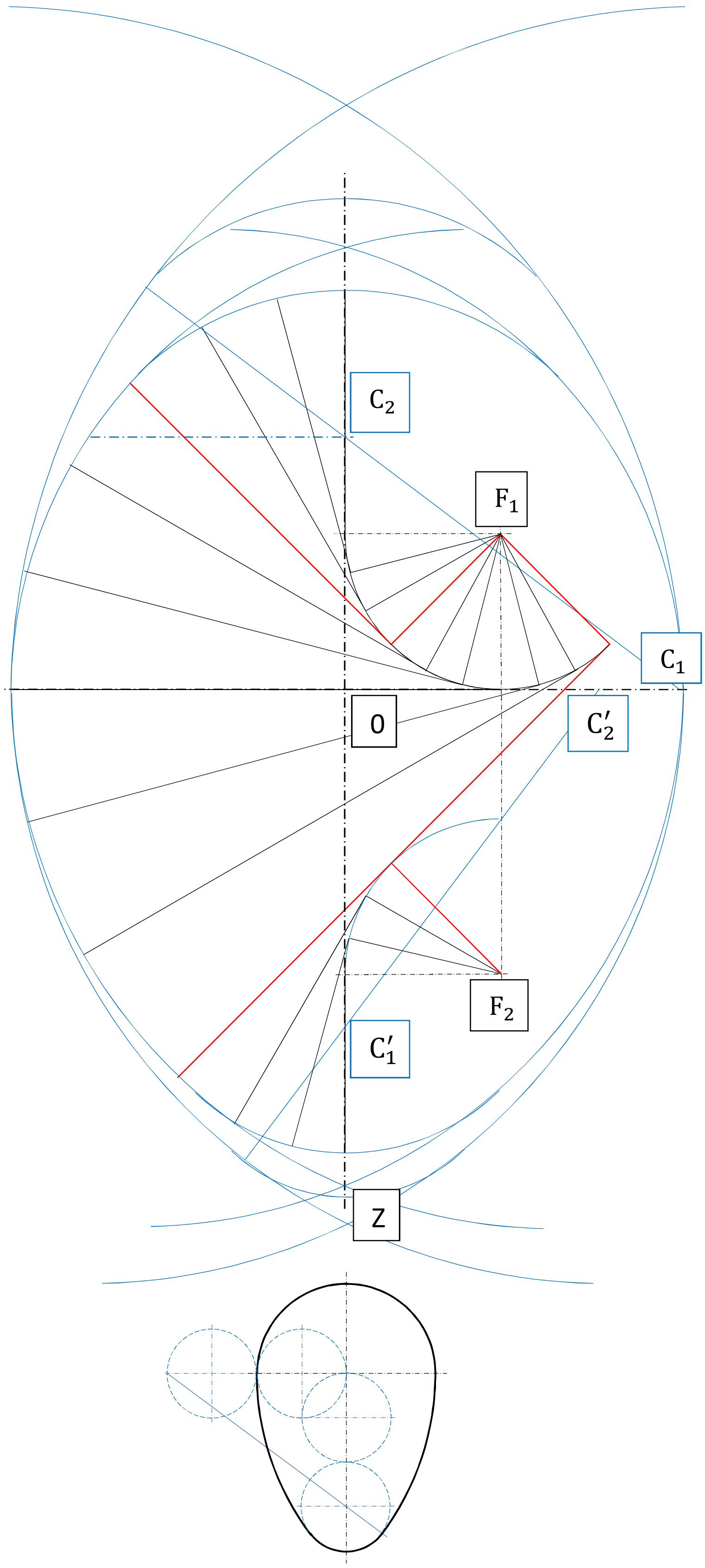


Fig.17

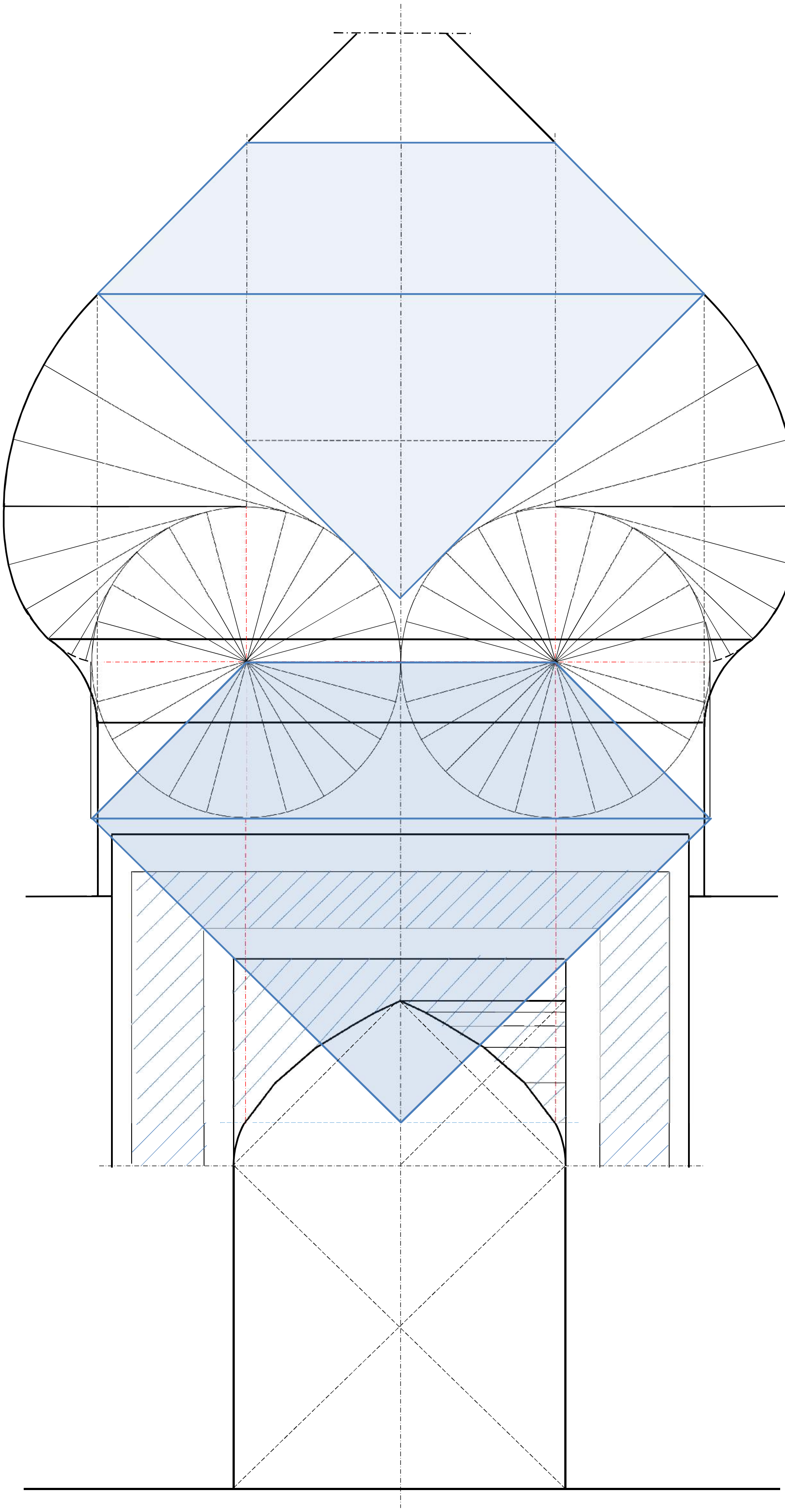


Fig. 18

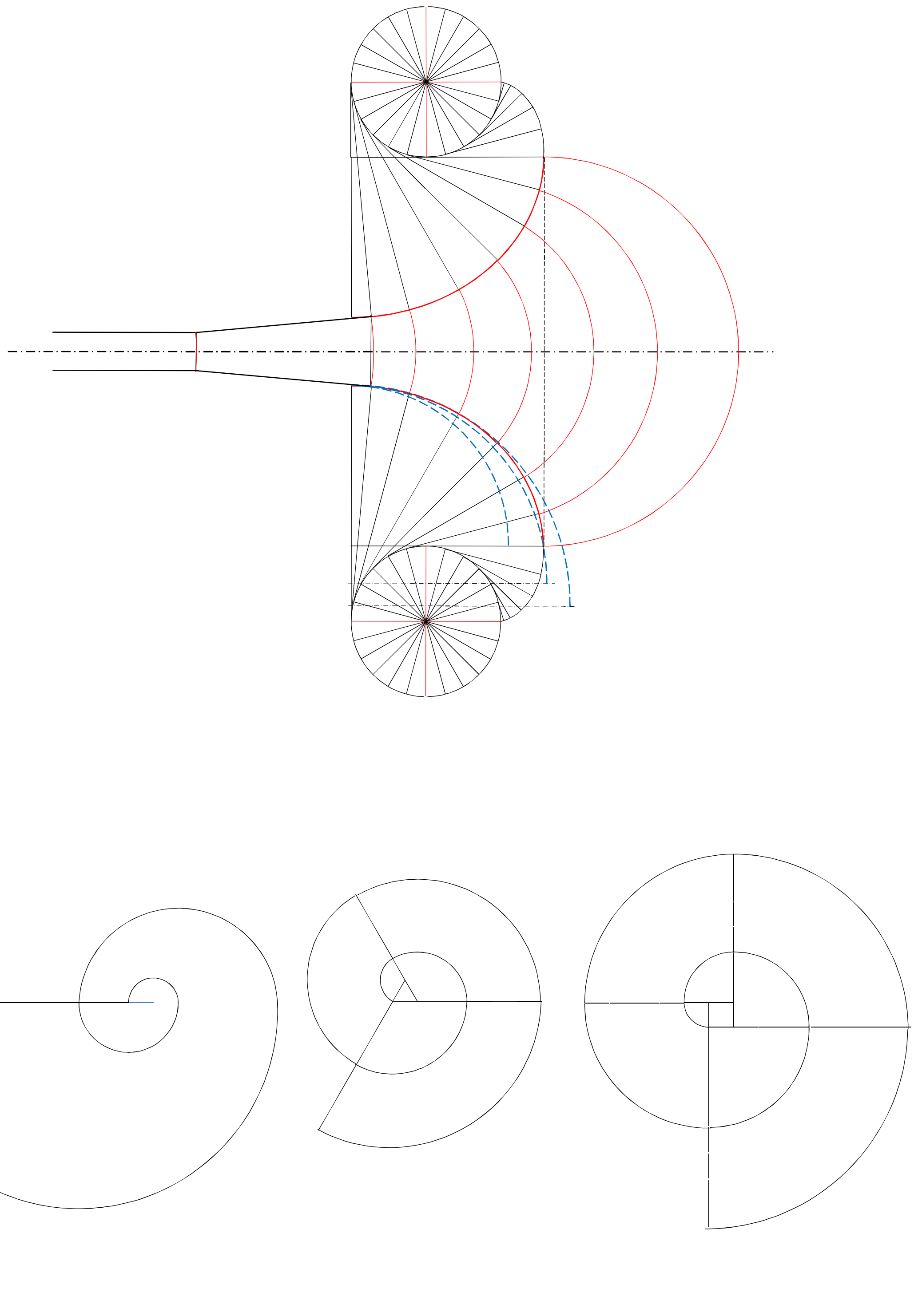


Fig.19