The full head vector in a free-diverging supersonic gas jet, and interaction of the same jet with diverging part of supersonic nozzle

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There is adduced example of modern comparative analysis of the state and motion parameters in free supersonic gas jet and at its interaction with diverging part of supersonic nozzle. It is showed, that in both these cases sonic velocity M = 1 is attained in expanding part of the jet but not in the converging nozzle neck. It is adduced method for physically adequate reproduction of the state and motion parameters in supersonic gas stream by means of traditional tables of gas dynamic functions.
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Introduction

Comparative analysis of the state and motion parameters in free supersonic gas jet and at its interaction with diverging part of supersonic nozzle is carried out on example of a photo of a visualized free supersonic air jet, presented by M.Je. Deitch in his book [1]. The jet outflowing out of narrowing nozzle under pressure drop, $p_h/p_0 = 1/20$, when linear dependence of gas flow rate on pressure drop is reached for the first time. The flow structure contains section of hydrodynamic acceleration of air stream in narrowing nozzle, $0.15 \le M \le 0.3$, and section of supersonic expansion and acceleration, M > 1, with external superlayer, which is springing up in transonic zone, $0.3 \le M = 1$. This superlayer is intensively accelerating and carries along internal part of stream; in the result convex profile of the stream velocity in narrowing nozzle turns into concave profile in supersonic jet.

Object of the given article is physically adequate elucidation of the state and motion parameters in free supersonic gas jet and its interaction with diverging part of supersonic nozzle.

Approach

The author uses graphic-analytical method as it is usual in his previous articles.

Solution

A known ratio

$$\boldsymbol{A_{cr}/A_n = (d_{cr}/d_n)^2},$$

where A_{cr} , A_n and d_{cr} , d_n are area and diameter of two adjacent sections of axisymmetric gas stream at M = 1and at outlet of converging nozzle respectively,

we compare with a known thermodynamic – isentropic – ratio of pressure in these two sections

$$\boldsymbol{A_{cr}/A_n = p_0/p_{cr}} ,$$

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where $$p_0/p_{cr} = [(k+1)/2]^{k/(k-1)} \quad and \quad k = c_p/c_v;$$

whence it follows $$d_{cr} = d_n[(k+1)/2]^{k/[2(k-1)]}.$$

In the result we come to equality of forces, moving gas stream in these two sections

$$p_0\, \pi d_n^2/4 = p_{cr}\pi\, d_{cr}^2/4,$$

and p_0 corresponds to the conic nozzle neck where M = 0.3.

Fig.1, in its left side, shows structure of a free supersonic air jet corresponding to the known results of Deitch's experiment, and in right side of this figure it is showed external profile of the jet in kind of a circumference evolvent as it is showed in previous article [2] of the author.
In the checking point, CP, we can construct a right-angled triangle with its hypotenuse tangent to evolvent and under 35°16' to the jet longitudinal axis. Displacement of the triangle down-stream of the jet leads to a decrease and then full disappearance of radial cathetus and at the same time leads to rapprochement and then to full coincidence of hypotenuse with axial cathetus. Such smooth transformation of right-angled triangle into a line parallel to the jet axis quite corresponds to our qualitative notion of decrease in static pressure and straitening of flow trajectories down-stream of a free supersonic gas jet.

Fig.2, in its right part, shows contour of supersonic nozzle with its diverging part in kind of the same circumference evolvent and checking point as it is showed in Fig.1. Now we can determine the state and movement parameters of supersonic gas stream by means of traditional tables of gas dynamic functions for isentropic flow. In the first place, we find radial component of a static pressure vector in checking point by means of the tables.
Then by means of *cos* 35°16' we determine a full static pressure vector; further we are copying contour of the nozzle supersonic part and carrying it up to contact with an end of the full static pressure vector. In that way we obtain distribution of the full static pressure vector along contour of the nozzle supersonic part (brown evolvent). Axial component of the full static pressure vector, directed against flow of gas stream, creates additional thrust onto wall of the nozzle diverging part. Certainly, in the case of free supersonic gas jet such additional thrust does not created.
Then, superposing line of the tables at M = 1 with checking point, CP, we obtain rest parameters of state and movement of supersonic gas stream in diverging part of the nozzle.

Discussion of results
Semi-empirical researches, carried out at the fiftieth and sixtieth of XX century, and mathematically strict approaches to a problem of optimum profiling of supersonic nozzle, carried out at the same times [3 – 8] and more later times, do not led to elucidation of questions both on type of curve for the profiling and on checking point M = 1 in diverging part of supersonic nozzle till now.

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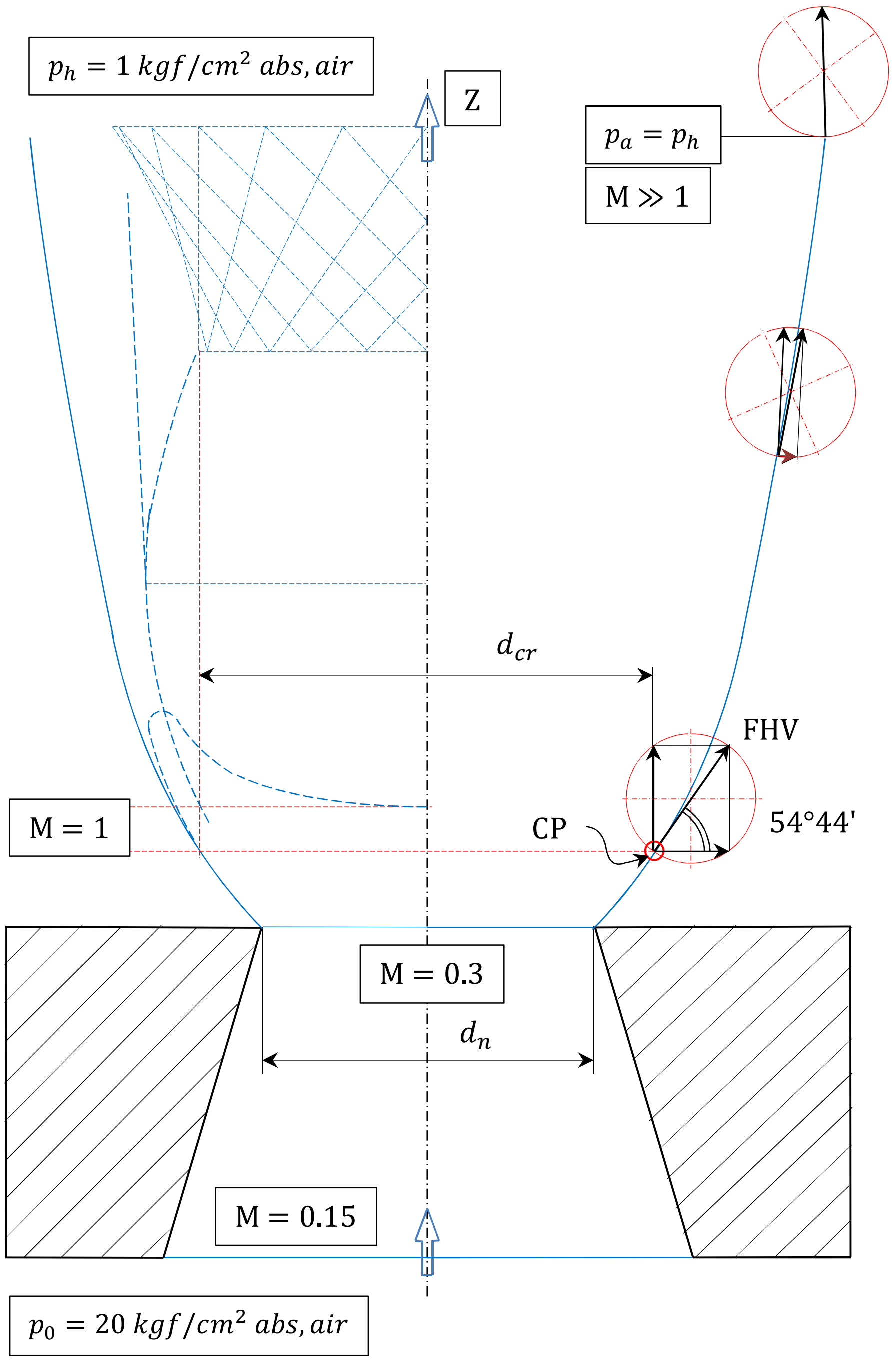

Fig.1: CP - *checking point,* FHV - *full head vector*

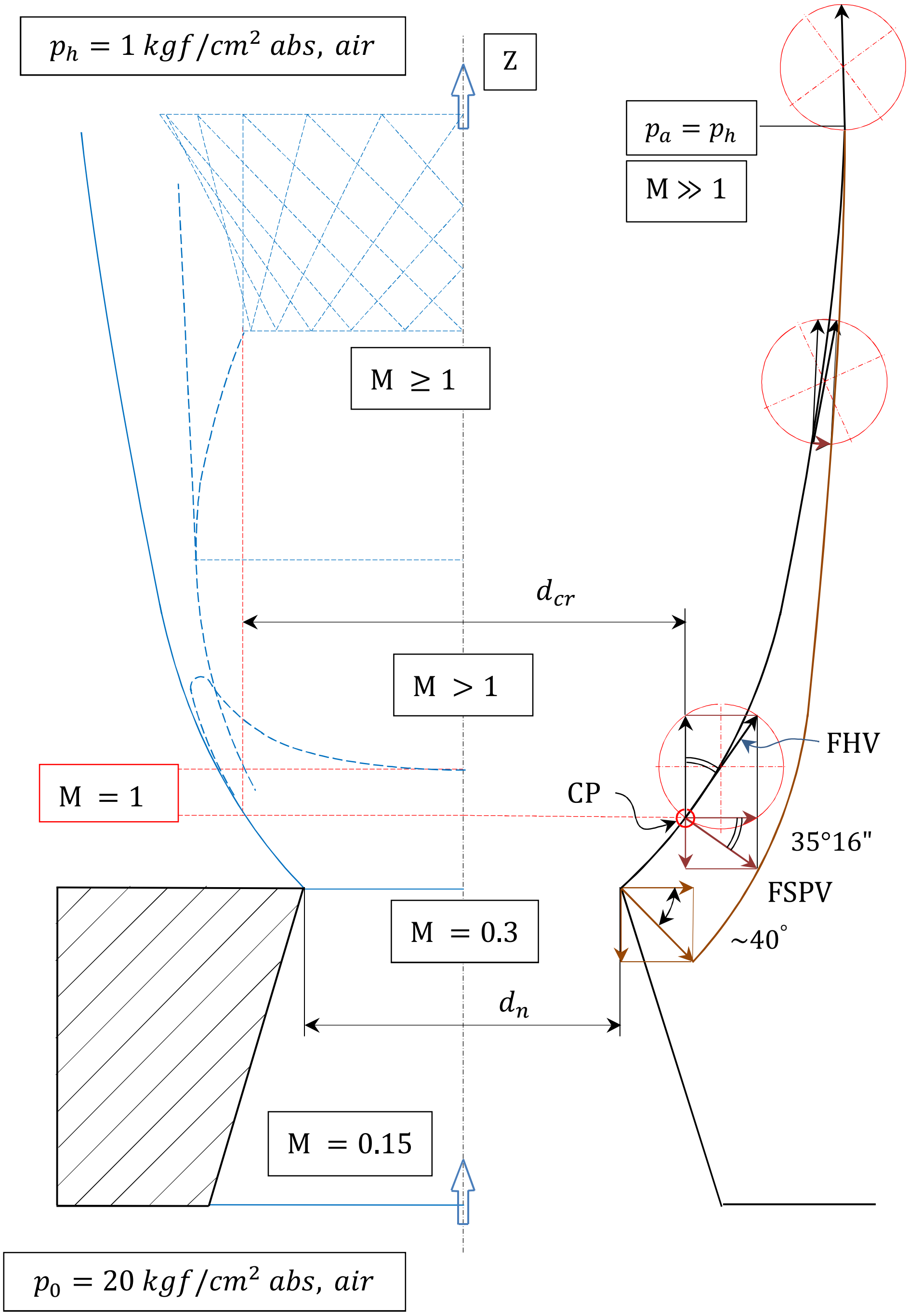

Fig.2: CP – *checking point,* FHV – *full head vector,*
FSPV – *full static pressure vector*