Modern fluid motion physics
Part 3: Saint-Venant – Wantzel formula modern form

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The modern form of the Saint-Venant − Wantzel formula for an outflow velocity of gas stream from flow element, system is presented. The taking into account of contact interaction of gas stream with the streamline surface in the form of the static head law has allowed finding the spatial-energy liaison in flow system. The physically correct combination of mechanics of contact interaction and thermodynamics of fluid in one formula has allowed simultaneously to be liberated from the velocity coefficient and the discharge coefficient and also polytropic process. In the new form the formula has gained the key character for computation of parameters of motion and state of gas stream in the flow system.

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Nomenclature

A − flow element cross-section area;
g − gravity acceleration;
H − general height of a fall;
h − height of the running point at free fall;
k − adiabatic exponent;
L − general length of the flow element;
p_0, p_h − pressure before inlet and after outlet of the flow element respectively;
$p_{st}(L)$ − static head along the fluid stream;
R − gas constant of the given gas;
T_0 − thermodynamic temperature of gas before inlet into the flow element;
V_{ex} − outflow velocity (average in the outlet cross-section);
V_{max} − maximum possible outflow velocity;
γ_0 − weight density of fluid before the flow element inlet.

Introduction

The development of modern ground works of gas dynamics in the field of flow elements and system is bound with necessity not only to create the physically adequate conceptual ideas and to derive on its base the new mathematical expressions, but also to revise and to find the final form of some existing formulas. The Saint-Venant – Wantzel formula for determining of the outflow velocity of gas stream out of flow element, system under specified value of pressure drop is one of the formulas, which one has major value for gas dynamics of flow systems and to which it is necessary to impart the final form.

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Approach

The considered formula is a product of long-term development. Its history was started by Torricelli and Galilei experiments with the drop water jets in 1643. Further, approximately 120 years later, Borda and Du Buat have given to it the final form for mechanics and hydraulics. After, about 60 years later (1839), Saint-Venant and Wantzel had substituted the available work of gas expansion, under conditions of mechanical and thermal isolation, instead of height of free fall in the Torricelli – Galilei – Borda – Du Buat (TGBD) formula. So, the Saint-Venant – Wantzel (SVW) formula was appeared

$$V_{ex} = \sqrt{2g \cdot \frac{k}{k-1} R T_0 [1 - (p_h/p_0)^{(k-1)/k}]}\,, \tag{1}$$

or

$$V_{ex} = \sqrt{\frac{2}{k-1} k g R T_0 [1 - (p_h/p_0)^{(k-1)/k}]}\,. \tag{2}$$

In the form (1), the SVW formula corresponds exactly to substitution in TGBD formula of the available work of gaseous medium expansion in reversible adiabatic – isentropic – process. In the form (2), the SVW formula is presented in technical literature as the outflow velocity of gas stream out of flow element, determined by a product of a maximum possible outflow velocity of gas – outflow into vacuum – and the dimensionless radicand, containing only initial, p_0, and final, p_h, pressures. In the contracted form, it corresponds to a record

$$V_{ex} = V_{max}\sqrt{1 - (p_h/p_0)^{(k-1)/k}}\,. \tag{3}$$

Having TGBD formula written in a form

$$V = \sqrt{2gH(1 - h/H)} \equiv V_{max}\sqrt{1 - h/H}, \tag{4}$$

it is not difficult to detect that H is simultaneously both difference of potentials of propulsive energy and spatial, longitudinal, coordinate of the motion during the free fall. At comparison of the TGBD and SVW formulas, it is not difficult also to note, that the spatial coordinate of the motion vanishes in SVW formula, when the available work of gas expansion in reversible adiabatic process is substituted instead of the height of free fall in TGBD formula. Such "feature" is entirely typical for thermodynamics; however, the usefulness of SVW formula becomes more than doubtful for calculation of the outflow velocity out of flow element. So, both viewed formulas are not bound with any kind of contact interaction of the moving solid, fluid with ambient bodies and mediums. And alongside with it the spatial coordinate of motion is absent in SVW formula. At first sight, the unfavorable situation becomes constructive at the approach to it from a position of the contact interaction energy. The SVW formula according to such approach is equation of the energy conservation (the weight or volume density more precisely) ion the simplest form. The problem is in that to take into account the spatially-energy connection of the flow system in this equation physically adequately and mathematically correct. Such mathematical expression for the spatially-energy connection of gas stream with a wall of the flow element is necessary, firstly, to have and, secondly, to know, how this connection to insert into the considered equation for the problem solution.

Solution

The mathematical expression of the spatially-energy connection of gas stream with a wall of the flow element is shown in previous articles [1, 2] in the form of static head law along the pipe flow element. In this law, there are simultaneously both the contact interaction energy and power of counterpressure of gaseous medium behind the outlet end of the flow element. This second factor has been presented in traditional form of SVW formula. The solution of the problem is in this situation reduced to replacement of p_h by $p_{st}(L)$ in initial SVW formula (1, 2). In a result we find

$$V_{ex} = \sqrt{\frac{2}{k-1} kgRT_0\{1 - [p_{st}(L)/p_0]^{(k-1)/k}\}}\,, \tag{5}$$

or

$$V_{ex} = V_{max}\sqrt{1 - [p_{st}(L)/p_0]^{(k-1)/k}}\,. \tag{6}$$

The weight flow formula according to it looks like

$$G = A\gamma_0(p_h/p_0)^{1/k} V_{max}\sqrt{1 - [p_{st}(L)/p_0]^{(k-1)/k}}\,. \tag{7}$$

Expressions (5, 6) are the final form of the formula for the outflow velocity of gas stream out of the flow element. The obtained expressions possess generality for the flow elements and systems, and initial SVW formula is its particular form for solution of the problems, concerned with description of the local gas point-symmetrical (spherical) expansion in unrestricted gaseous medium, when a mass center of the gas is kept in rest.

Discussion of results

The obtained expression in the forms (5, 6) implies, that the actual outflow velocity of gas stream as kinematic parameter of motion is determined by its maximum possible quantity, V_{max}, at outflow into empty space with taking into account of energy contents of the outflowing and surrounding compressible fluids and by intensity of a contact interaction of gas stream with the wall of the flow element. Such physical sense is much wider than the sense, concluded in it by Saint-Venant and Wantzel and consisting in the simple substitution of expression for available work of gas expansion at reversible adiabatic – isentropic -- or polytropic process instead of height of free fall in TGBD formula. The new physical sense is wider also than traditional thermodynamic interpretation, according to which the outflow velocity is determined only by difference of heat contents – enthalpies – of outflowing and surrounding compressible fluids. The modified formula distinguishes in principle also in that the gas motion is presented in it as mechanically irreversible process. This property of a gas motion in the flow element is the alienable and permanent factor. The heat exchange can be absent; in this case, the flow process will be adiabatic; it can be absent action on the gas stream of any physical factors: heat, additional weight flow, technical work and others. A factor of the contact interaction of the gas stream with the flow element wall acts always and renders primary and determining influence on the gas stream state and its motion parameters. Expansion of physical sense of the considered formula from narrow-thermodynamic to the full value thermo-mechanical allows utilizing it as effective tool for physically adequate analysis of the real gas motion process and its subsequent mathematically correct calculation of the motion and state parameters in multiplicity of actual circumstances that is, with taking into account of a contact interaction, heat transfer, additional weight flow, technical work, the aggregate state modifications, thermo-chemical transmutations. At comparing the obtained formula with initial SVW formula, it is necessary to note, that they both allow to determine the gas outflow velocity, firstly, at outlet of the flow element (pipe) and, secondly, as the average weight flow quantity that is without taking into account of features of its distribution in the flow element cross-section. Along with it, it is necessary to note, the difference in principle between them. Firstly, the new formula allows taking into account the influence onto the outflow average velocity of any physical factors, acting along the way of gas stream motion. Secondly, the new formula is free from "magic" influence to it of such conventional constant as the critical ratio of pressures. It itself allows determining the individual critical ratio for any actual flow element with taking into account of physical factors, acting onto the stream. It is necessary also to note that the attempts of usage of the SVW formula for solution of technical problem in the gas dynamics have been led to a number of its "perfections" during last approximately 120 years.

The velocity and weight flow coefficients were, in partially, entered into the "applied" gas dynamics by analogy with hydraulics [3]. However, experiments have shown that, in contrast to hydraulics, the quantity of the weight flow coefficient for the real gas stream can be both smaller and greater than unity. In one of the correctly realized experiments [4], the discharge of vessel happened much faster than in the results of calculation by means of isentropic SVW formula.

At the second half XIX century, technical thermodynamics was enriched by invention of polytropic process as generalization of laws of the gas states. The generalization was attained by means of replacement of adiabatic exponent by polytropic exponent, variable in the limits $0 \leq m \leq \infty$ [3, 5 and 6]. Let alone that, what this exponent requires experimental determination for each special case, this idea in itself is in full contradiction to the molecular-kinetic theory.

Introduction the isentropic process conception, entirely appropriate to SVW formula, has led to creation of the gas dynamic tables in the form of system of the ratios, devoid of longitudinal coordinate of the gas stream motion [7].

Not stopping on more small-sized "betterments", it is necessary to underline, the expressions (5, 6 and 7) allow correctly and precisely to present the motion of the real gas stream in actual circumstances and ones does not require changes at determining the flow parameters both under subcritical and under supercritical conditions.

Final remarks

Only now, after 164 years, it is obtained the Saint-Venant – Wantzel formula form, allowing in combination with the static head law [1, 2] for the first time rightly and exactly to determine the state and motion parameters of real gas in its actual motion in the flow element, system.

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