Modern fluid motion physics: the gas equation for stream

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The equation of state for a gas stream as analog of well-known equation of state of gas medium under static conditions is presented. Connection between the state and motion parameters and the weight- and volume-flow rates of the gas stream in the flow system during change of its temperature and pressure as the result of external action is shown on example of modification of the represented equation.

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The history of development of gas equation, well-known by B. Clapeyron (1834) and D. Mendeleyev (1874) names, includes more than the 200-years period (1661-- 1874). However, the deep sense and significance of this law can be understood only now, at the close of the XX and at the beginning of the XXI centuries, owing to the solution one of the classical problems of physics -- development of the conceptual apparatus for description and quantitative evaluation of state and motion of gas stream and for simulation of fluid motion in general. This law, which generalizes the special gas laws, was determined methodologically in the context of consideration of the continual gas medium and phenomenologically -- on the basis of direct experiment. Thus the level of generalizing in this law is determined, in not a smaller degree, also by reflection in it of the mechanical equivalent of heat in the form of the gas constant (universal or specific) and connection with quantity of the substance.

It is remarkably, that three special gas laws: R. Boyle's and E. Mariotte's, J. Charles', Charles' and J. Gay-Lussac's -- is established historically in sequence from a simple to a complex.

The importance of the first gas law, established by R. Boyle (1661) and E. Mariotte (1676), $p_1V_1 = p_2V_2$ or $pV = const$, along with its quantitative expression, consists in fundamentality of the approach, associated with concept of a compressibility as the property of bodies and mediums to change its volume and form under loading and elasticity as ability to recover initial volume and form at unloading. Besides, the conditions of holding of experiment, $T = const$, ensured the greatest universality of the approach to the test object. As Boyle wrote himself: «Both Pascal's experiments and experiments of our English friend (R. Hooke) have demonstrated that the greater weight acts to the air, the more strong becomes its tendency to expansion and, therefore, its resisting strength (similarly to spring plates, which ones become more strong, when ones undergo bending by the larger loads).» The approach is one both to the air, and to the spring plates, $T = const$. The fundamentality of Boyle's conceptual system, including such categories as compressibility and elasticity, undoubtedly, has formed the basis for determination of Hooke's law of elasticity conformably to a tension of solid bodies.

The first law, as well as both following special gas laws, was determined by means of experiment with the fixed quantity of the gas, placed in a vessel, hermetically sealed by piston. In this connection, all three special gas laws are applicable to the so-called closed-loop systems with fixed amount of gas medium.

The Boyle experiment, conducted under stationary temperature, characterizes the *isothermal* process of the pressure and volume change of the given amount of gas. The *isothermal* change, in modern thermodynamic interpretation, is implemented at heating up of gas with a simultaneous external action (its expansion), at which all heat, injected toward gas, is compensated by an expansion work ensuring an invariability of the reference temperature of the

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gas and its internal energy.
Later 15 years after Boyle's experiments, the first special gas law was established by Mariotte on the basis of self-maintained vast investigations predominantly of gas medium (although the Mariotte's formula for an estimate of stresses in a thin-wall cylindrical shell under action of pressure of fluid medium also occupies the substantial place in the strength theory).
The second special gas law was established by Charles (1787), and in two forms of its writing $p_2 = p_1(1 + \alpha_p t_2)$ and $p_1 T_1 = p_2 T_2$ or $p/T = const$, it expresses the one aspect: in the closed vessel of invariable volume, the heating up of gas leads to its pressure increase so, that the ratio of the pressures is equal to the ratio of temperatures. The Charles law, as well as the first special gas law, is an evidence of reversibility of behavior of gas, that is, of its elasticity.
Process, which realizes in gas medium under invariable volume, is called the *isochoric* process, in modern thermodynamic interpretation. The constancy not only volume of gas, but also its amount and, ultimately, of its weight density is typical for such process. In this process, all added heat is expended onto heating of gas, that is, on increase of its internal energy.
The third special gas law was established, as well as first law, twice and with the same difference in years: by Charles (is not published in time), in 1787, and by Gay-Lussac, later 15 years, in 1802, and one also has two forms of its writing similarly to the second law:
$V_2 = V_1(1 + \alpha_V t_2)$, where $\alpha_V = 1/273\ K^{-1} = \alpha_p$ and $V_1/T_1 = V_2/T_2$ or $V/T = const$.
This law implies, that for a given and invariable amount of gas in the vessel, the volume of which can be changed, the heating up of gas leads to an increase of its volume so, that the ratio of volumes is equal to the ratio of temperatures under constant pressure. Here again, the gas medium behaves oneself elastically, as in two previous laws. It is typical for *isobaric* process, in modern thermodynamic interpretation, that at heating up of invariable amount of gas is possible to ensure constancy of pressure in it by the resolution of free (thermal) expansion.
Gay-Lussac was attempting to unify the special gas laws in one generalized law, taking into account of evident connection of effective parameters, T, p, V, and the unity of the tested medium. In a result he has received expression $p \cdot V/T = const$, in 1826. It is attested, that the phenomenological character of the special laws does not contain some conceptual component, permitting to uncover sense and magnitude of constant in the right member of the equation.
The following attempt of a deduction of a generalized gas law belongs to B. Clapeyron. He has received $pV = BT$ (1839) and then, in 1840, $p \cdot V/T = BM$, where B is a proportionality constant and M -- weight of gas.
The opportunities of deriving of the generalized gas law by the results of experiments with gas medium and by means of phenomenological approaches to the problem were exhausted.
In the beginning of the second half the XVIII century, J. Black has clearly explained the law of thermal equilibrium in parallel with the above-mentioned investigations. He has introduced the notion of heat capacity of bodies and mediums. F. Delaroche and J. Berard have realized the first precise measurements of specific heats of gases in 1760. The results of these experiments were promoting development of ideas about a possibility to make the work by gas at the expense of its internal energy at absence of thermal action from the outside. P. Laplace (1816), comparing heat capacities of isochoric by Charles and isobaric by Charles and Gay-Lussac processes, has determined their inequality in that in isobaric process the heat capacity of gas is bigger in $k = c_p/c_V$ times. Laplace has resulted to the final form the formula for a sound speed in gas medium, that is, from the form $= \sqrt{p/\rho}$, previously deduced by I. Newton on the basis of the Boyle law and the E. Torricelli formula, to a form $a = \sqrt{k \cdot p/\rho}$ in the same year, 1816.
The invention of the pneumatic fire, in France in 1803, and the Gay-Lussac experiments by compression and expansion and also by transfusion of gas between the two heat-insulated vessels have resulted to the concept on adiabatic process, in 1807, and to linkage of thermal energy with work of the expansion and compression of gas. In the result, S. Poisson (1823) has deduced the equation of adiabatic process in the form $pV^k = const$, where $k = c_p/c_V$.

Investigation of properties of adiabatic process and the knowledge of the Newton and Laplace approaches to the Torricelli formula have allowed to B. Saint-Venant and P. Wantzel (1839) deducing the formula for the outflow velocity of the gas medium out of a space under high pressure p_0 into a space of the same gas medium under low pressure p_h

$$V_{ex} = \sqrt{2g \cdot \frac{k}{k-1} RT_0 [1 - (p_h/p_0)^{(k-1)/k}]}.$$

The analysis of this formula has allowed elucidating such substantial features of gas motion, as baro-subcritical and baro-supercritical outflow and outflow into empty space. However, given formula has not wished to feature flow of gas in the real flow element, and especially in the flow system, even in absence of heat exchange and other physical actions. The coefficients of velocity and the mass-flow rate was added to it as it is accepted in hydromechanics, adiabatic process was replaced by the polytrope -- all in vain. Only today, later 150 years, the belladonna-formula has uncovered its true sense [1].

In the first half the XIX century, the development of the elementary bases of thermodynamics was going on parallel with building of knowledge on the gas laws. J. Mayer (1842) search out the expression for mechanical equivalent of heat, $c_p - c_V = R$, which is equally indispensable for generalizing the gas laws and for development of thermodynamics, and still 32 years later, Mendeleyev has completed a history of making of gas equation, including the mechanical equivalent of heat and the Kelvin temperature side by side with the special gas laws

$$p = \frac{G}{V} RT$$

This appearance has passed quietly enough and almost not influenced to the problem of the gas motion. At all events, the scientific association has understood that this equation not results in the radical improvement in the gas dynamics both in itself and in combination with the fundamental Torricelli -- Saint-Venant -- Wantzel formula and in combination with the formal mathematical line, and one was perceived enough indifferently. And 120 years later, it turned out, that the given generalized equation requires development with the purpose of its application for description of gas stream motion.

This equation is rather productive conformably to the concrete circumstances. At the same time, it is very important whether we understand physics of processes in gas stream and significance of this equation for its description. It expressed condition of state of motionless gas medium under thermal action in a closed-loop system. As applied to open systems, for example, in the form of the pipe flow element, we gain again three special gas laws, having accepted the weight- and volume-flow rates of gas stream instead of weight and volume of motionless gas respectively

$$T = \frac{1}{R} \cdot \frac{Q}{\dot{G}} p \equiv \frac{1}{R} \cdot \frac{F \cdot w}{\dot{G}} p, \tag{1}$$

$$Q = \frac{G\dot{R}T}{p} \quad or \quad w = R \frac{1}{F} \dot{G} \frac{T}{p}, \tag{2}$$

$$p = R \frac{\dot{G}}{Q} T \equiv R \frac{1}{F} \cdot \frac{\dot{G}}{w} T, \tag{3}$$

where *F* area of cross-section of the flow element, *w* -- velocity of gas flow, Q and $\dot{G}$ -- the volume- flow rate and the weight-flow rate through the flow element accordingly.

Operating by analogy with the special gas laws for the closed system we shall receive

$$T = \frac{1}{R} \cdot \frac{Q}{\dot{G}} p \equiv \frac{1}{R} \cdot \frac{F \cdot w}{\dot{G}} p = const; \qquad (1a)$$

$$Q = \frac{\dot{G}R}{p} T = const \;\; or \;\; w = \frac{1}{F} \cdot \frac{\dot{G}R}{p} T = const; \qquad (2a)$$

$$p = \frac{\dot{G}R}{Q} T \equiv R \frac{1}{F} \cdot \frac{\dot{G}}{w} T = const. \qquad (3a)$$

The obtained dependences are valid for the stationary steady flows of gas medium and ones allow to establish the type of the process realized in gas flow, depending on the separate or combined action of heat flow and pressure onto gas stream. The action by pressure, in this case, means its increase or decrease simultaneously and equally in spaces, surrounding the inlet and outlet of the flow element, system. According to the dependence $(1a)$, the increase or decrease of the pressure in the above indicated spaces calls change of correlation of the volume-flow rate and the weight-flow rate, compensating the action of pressure so, that the isothermal change is ensured in gas stream. For example, the gas stream weight-flow rate in flow element, system under action of increased pressure is incremented, and the volume-flow rate (velocity) of a stream is diminished ensuring an invariability of temperature of gas in a stream, i.e. it's the isothermal state. According to the dependence $(3a)$, the action with heat flow to gas stream is accompanied by respective change of correlation of the volume-flow rate and weight-flow rate so, that the isobaric process is ensured in a gas stream. For example, the gas stream weight-flow rate under action of heating is diminished and volume-flow rate (velocity) of gas stream is being increased, ensuring an invariability of pressure in gas stream, i.e. it's the isobaric state. According to the dependence $(2a)$, the change of weight-flow rate happens so, that the volume-flow rate (velocity) of stream is remained fixed under action simultaneously both of heat flow and pressure on gas stream. For example, the volume-flow rate and the weight-flow rate (velocity) will remain invariable under heating of stream and the simultaneous equivalent recompression in spaces, surrounding the flow element, system. The lowering of weight-flow rate will happen if the thermal factor will predominate over the pressure, and the volume-flow rate (velocity) of stream will remain former in this case. If the factor of pressure will predominate over the thermal action, the weight-flow rate of stream will increase exactly so, as far as it is necessary that the volume-flow rate (velocity) of stream has remained invariable. In any of these cases, the analogy of state of gas stream with isochoric process in static conditions is remained in the kind of dependence $(2a)$.

As a whole three special laws of gas state in stream is integrated in one expression

$$p = \frac{\dot{G}}{Q} \cdot RT, \qquad (4)$$

being dynamic analogue of the Clapeyron-Mendeleyev equation and indispensable component for model operation of motion and state of gas stream under action of temperature and pressure.

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[1] S.L. Arsenjev, Modern fluid motion physics: Saint-Venant -- Wantzel formula modern form, 2003